Problem Set 4 – Due Thursday, October 25, 2012

Problem 1. Let α be a regular expression of length n.

(a) Using procedures shown in class, if we convert α into a regular expression β such that $L(\beta) = \overline{L(\alpha)}$, how long might β be? Give a reasonably tight upperbound.

(b)* Can you define an infinite family of regular expressions $\{\alpha_n\}$, $|\alpha_n| \in O(n)$, but where the shortest regular expression for $\overline{L(\alpha_n)}$ will have length $\Omega(2^n)$?[1]

Problem 2. Using the pumping lemma, show that the following languages are not regular.

(a) $L = \{www : \ w \in \{a, b\}^*\}$.

(b) $L = \{a^{2^n} : \ n \geq 0\}$.

(c) $L = \{0^n 1^m 0^n : \ m, n \geq 0\}$.

Problem 3. Define $A = \{x \in \{a, b, \sharp\}^* : x$ contains an equal number of a's and b's or x contains consecutive $\sharp$s or consecutive letters$\}$.

(a) Can you use the pumping lemma to prove that A is not regular? Explain.

(b) Prove that A is not regular.

Problem 4. Are the following statements true or false? Either prove the statement or give a simple counter-example.

(a) If $L \cup L'$ is regular then L and L' are regular.

(b) If L^* is regular then L is regular.

(c) If LL' is regular then L and L' are regular.

(d) If L and L' agree on all but a finite number of strings, then one is regular iff the other is regular.

(e) If R is regular, L is not regular, and L and R are disjoint, then $L \cup R$ is not regular.

(f) If L differs from a non-regular language A by a finite number of strings F, then L itself is not regular.

Problem 5. Carefully describe an algorithm to answer the following question: given a regular expression α, is $L(\alpha) = (L(\alpha))^R$? What is the asymptotic running time of your algorithm?

Problem 6. For any language L let

$$\mathcal{F}(L) = \{w \in L |\text{ no proper prefix of } w \text{ is a member of } L\}$$

Prove or disprove: the regular languages are closed under $\mathcal{F}$.

[1] This problem is intended for, at most, the top students. If you can find an elementary solution without consulting the literature, please give it directly to Prof. Rogaway.