Problem Set 6 – Due Thursday, November 8, 2012

For this problem set, you may work in groups of 1–4 people. Please turn in only one problem-set solution per group. Make sure to list all names of all group members on the writeup. If you understand this stuff well, please be kind enough to try to find a group that needs you (as opposed to being a "group" of size one).

I strongly encourage you to work out problems 1–3 before your midterm; just consider it part of your midterm-prep. You won't be able to do Problem 4 until just after the midterm.

Discussion section on Wednesday will be "facilitated small-group interaction." There won't be any sort of lecture, but show up to find a group with which to finish this problem, or to engage in other discussions on ECS 120 and the Meaning of Everything.

Problem 1. Using the pumping lemma, prove that $L = \{b_i \# b_{i+1} : b_i \text{ is } i \text{ in binary}, i \geq 1\}$ is not context free.

Problem 2. Alice tries to prove that the language $L = \{1^i + 1^j = 1^{i+j} : \quad i, j \geq 0\}$ is not context free using the pumping lemma.[1] Alice assumes for contradiction that L is context free and lets "p" be the pumping length for L as guaranteed by the pumping lemma. Alice lets s be the string $1^p + 1^p = 1^{2p}$. The string s is in L and has length at least p, so the pumping lemma tells us that s can be partitioned into $uvxyz$ where $|vy| \geq 1$ and $|vxy| \leq p$ and $uv^ixy^iz \in L$ for all $i \geq 0$.

Try to finish Alice's proof. Does any case give you trouble? If so, which? Is it *possible* to prove that L is not context free?

Problem 3 Read problem 2.18 of your book. as well as the solution (a few pages later) that Sipser provides.[2] Then write an English-language paragraph that clearly explains, in your own words and without a lot of symbols, why it is that the intersection of a regular language and a context-free language is always context free.

Problem 4 Turn in a perfect solution for Tuesday's midterm. (I will post the midterm just after you finish taking it.)

[1] Here "+" and "=" are just characters of the alphabet Σ over which strings from L are drawn.
[2] The problem is identical in the second and third editions of the book.