

Problem Set 7 – Due Thursday, November 15, 2012

For this problem set, please work in teams of 2–3 people. Submit one solution per team.

Problem 1. Design a Turing machine that decides the language

$$L = \{ww : w \in \{0, 1\}^*\}.$$

Rather than following the conventions of your book, please employ those of the website <http://morphett.info/turing/turing.html>. In particular, assume a two-way infinite tape and, instead of accepting or rejecting by entering a designated state, have your machine print 1 (accept, yes) or 0 (reject, no) on an otherwise blank tape. Try to make your program use as few rules as possible, measured by the number of 5-tuples that you need. Test your machine on plenty of inputs both in and not in L . A prize will go to the (apparently correct) solution with the fewest number of rules.

For grading this problem, please mail your solution, in the runnable format of the website above, to hbzhang@ucdavis.edu. A comment at the top of your program should list the names of the team members, in alphabetical order by last name, and the number of rules you used. A student solution will be used for our problem-set solutions.

Problem 2. A TM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R)$ is **oblivious** if, for all equal-length $x, x' \in \Sigma^*$, the position of the head at time t when started in configuration q_0x is the same as the position of the head at time t when started in configuration q_0x' . (It follows that if one machine halts on input x then the other halts on input x , using the same number of steps.) Prove the following: for any TM M there exists an oblivious TM M' that does decide, and accepts, the same language: M' accepts x iff M accepts x , and M' rejects x iff M rejects x .

Problem 3. Give an unrestricted grammar¹ for the language L of problem 1. Make your grammar as simple to understand as possible. Explain how it works in a clearly written English paragraph.

¹An unrestricted grammar $G = (V, \Sigma, R, S)$ is like a CFG except that rules can look like $aBCb \rightarrow ccdDe$, for example: the left-hand side can contain any string of terminals and nonterminals as long as it has at least one nonterminal. (Formally, R is a finite subset of $(V \cup \Sigma)^*V(V \cup \Sigma)^* \times (V \cup \Sigma)^*$.) If $\alpha_1 \alpha_2$ is a sentential form and $\alpha \rightarrow \beta \in R$ is a rule then $\alpha_1 \alpha \alpha_2 \Rightarrow \alpha_1 \beta \alpha_2$. As usual, $L(G) = \{x \in \Sigma^* : S \xrightarrow{*} x\}$.