

## Problem Set 9 — Due Thursday, December 6

**Problem 1.** State whether the following claims are true or false, briefly explaining your answer.

- a.  $A \leq_P A$ .
- b. If  $A \leq_P B$  and  $B \leq_P C$ , then  $A \leq_P C$ .
- c. If  $A \leq_P B$  then  $\overline{A} \leq_P \overline{B}$ .
- d. If  $A \leq_P B$  and  $B$  is decidable then  $A$  is decidable.
- e. If  $A \in P$  then  $A \leq_P a^*b^*$ .
- f. If  $A$  is r.e., then  $A \leq_P A_{TM}$ .

**Problem 2.** Suppose you are given a polynomial time algorithm  $D$  (for “decision”) that, on input of a boolean formula  $\phi$ , decides if  $\phi$  is satisfiable. Describe an efficient procedure  $S$  (for “search”) that *finds* a satisfying assignment for  $\phi$ . How many calls to  $D$  does  $S$  make?

**Problem 3.** Let  $MULT\text{-}SAT = \{\langle \phi \rangle \mid \phi \text{ has at least ten satisfying assignments}\}$ . Show that  $MULT\text{-}SAT$  is NP-complete.

**Problem 4.** A graph  $G = (V, E)$  is said to be  $k$ -colorable if there is a way to paint its vertices using colors in  $\{1, 2, \dots, k\}$  such that no adjacent vertices are painted the same color. When  $k$  is a number, by  $k$ COLOR we denote the language of (encodings of)  $k$ -colorable graphs. The language 3COLOR is NP-complete. (You can assume this.) Use this to prove that the language 4COLOR is NP-complete, too.

**Problem 5.** *Problem 7.26 from your book, where a supporting picture can be found.* You are given a box and a collection of cards. Because of the pegs in the box and the notches in the cards, each card will fit in the box in either of two ways. Each card contains two columns of holes, some of which may not be punched out. The puzzle is solved by placing all the cards in the box so as to completely cover the bottom of the box (ie, every hole position is blocked by at least one card that has no hole there). Let  $PUZZLE = \{\langle c_1, \dots, c_k \rangle \mid \text{each } c_i \text{ represents a card and this collection of cards has as solution}\}$ . Show that  $PUZZLE$  is NP-complete.

**Problem 6.** Let

$$D = \{\langle p \rangle : p \text{ is a polynomial (in any number of variables) and } p \text{ has an integral root}\}.$$

Prove that  $D$  is NP-hard. Is it NP-complete?