Problem Set 1—Due Friday, October 6

Instructions: Do all 5 problems. Write up your your solutions as clearly and succinctly as you can. Don't forget to acknowledge anyone with whom you discussed problems.

Problem 1. Page 28, Problem 2.

Problem 2. Let $G = (V, E)$ be a graph[1] on vertices $V = (v_1, \ldots, v_n)$. The *degree sequence* of G is the sequence of numbers $(d_1, \ldots, d_n)$, where $d_i = d(v_i)$ is the degree of vertex v_i.[2] Prove the following:

Part a. If G is a graph with degree sequence $(d_1, \ldots, d_n)$, then $\sum_{i=1}^{n} d_i$ is even.

Part b. If $(d_1, \ldots, d_n)$ is a sequence of non-negative integers with $\sum_{i=1}^{n} d_i$ even, then there is some graph G which has $(d_1, \ldots, d_n)$ as its degree sequence.

Problem 3. Page 74, Exercise 3.

Problem 4. Page 74, Exercise 4.

Problem 5. Page 76, Problem 7.

[1] Not necessarily *simple*—allow "loops" (edges (v, v)) and multiple edges joining a given pair of vertices.

[2] Remember that the degree of a vertex is the number of edges which touch it, each loop counting as two edges.