Problem Set 5—Due Friday, November 3

Problem 22. *(Postponed from last week.)*

Part A. Prove that $L_a = \{a^i b^j c^k : j = \max\{i, k\}\}$ is not context free.

Part B. Prove that $L_b = \{b_i \# b_{i+1} : b_i \text{ is } i \text{ in binary}, i \geq 1\}$ is not context free.

Problem 23. Is the following language context free? Prove your answer either way.

$$L_c = \{ww^R w : w \in \{a, b\}^*$$

Problem 24.

Part A. Prove that the intersection of two context free language is not necessarily context free.

Part B. Prove that the intersection of a context free language and a regular language is context free.

Problem 25. A *left/right automaton* is like a DFA except the head can move to the right *or* the left—which way it moves is dictated by the machine's finite control. The input tape remains read-only. The input $x \in \Sigma^*$ is presented to the machine delimited by special characters, $\square x \square$. The machine *accepts* its input whenever it wants to.

Carefully and *rigorously* formalize the notion of a (deterministic) left/right automaton M. Formalize the class of languages recognizable by left/right-automata. Make a (true) *conjecture* about the class of left/right-automata recognizable languages.

Problem 26. An *overwrite automaton* is identical to a left/right automaton except that now the machine can write as well as read any of the $|x|$ cells on which the input $x \in \Sigma^*$ is presented. The alphabet of characters written to these tape squares is some finite $\Gamma \supseteq \Sigma$.

Prove that overwrite automata accept a *proper superset* of the context free languages.[1]

[1] *Hint*: For context-free grammar G, bound the length of the longest sentential form in some derivation of $w \in L(G)$.