Problem Set 1 — Due Thursday, 8 April 2004

Instructions: Write up your solutions as clearly and succinctly as you can. Typeset solutions, particularly in LaTeX, are always appreciated. Don't forget to acknowledge anyone with whom you discussed problems. Recall that homeworks are due at 4 pm on Thursdays in the turn-in box in Kemper, on the second floor.

Problem 1. The following question is to remind you about **inductive definitions** and their use.

- **A.** Give an inductive definition for a *decimal number*. A decimal number is a string over the alphabet $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots, \mathbf{9}\}$. Examples are **4**, **120**, **007**.
- **B.** Give an inductive definition for the *value* of a decimal number. This is a map val from decimal numbers to nonnegative integers.
- **C.** Write a recursive function, in C/C++, to compute val. Assume that $\mathsf{val}(x)$ fits into a type `int`.

Problem 2. For each of the following, give an example language to prove existence, or explain why there is no such example. Assume an underlying alphabet of $\{0, 1\}$.

- **A.** An infinite language L with an infinite complement.
- **B.** A language L closed under concatenation and containing no even-length strings.
- **C.** An infinite unary language L such that if $x \in L$ and $y \in L$ then there is no string in L of length $|x| + |y|$. (A unary language means that the underlying alphabet has just one character, say **1**.)
- **D.** A finite language L having a longest string x that is longer than a longest string of any other finite language.
- **E.** Infinite languages L_1, L_2, L_3 with empty (three-way) intersection, but any two of which union to make $\{0, 1\}^*$.
- **F.** An infinite language L such that, for every number n, L contains strings x and X where $|X| - |x| > n$ and L contains no string y where $|x| < |y| < |X|$.

Problem 3. How many different languages over the alphabet $\{\mathbf{1}\}$ are accepted by two-state DFAs? By three-state DFAs? In each case, name them all.