Problem Set 2 — Due April 15, 2004

Problem 1. Give DFAs for the following languages. Assume the alphabet for the DFA is $\Sigma = \{0, 1\}$.

(a) The set of all strings with 010 as a substring.

(b) The set of all strings which do not have 010 as a substring.

(c) The set of all strings which have an even number of 0's or an even number of 1's.

(d) The complement of $\{1, 10\}^*$.

(e) The binary encodings of numbers divisible by 3: $\{0\}^* \circ \{\varepsilon, 11, 110, 1001, 1100, 1111, \ldots\}$.

Problem 2 State whether the following proposition are true or false, proving each answer.

Part A. Every DFA-acceptable language can be accepted by a DFA with an even number of states.

Part B. Every DFA-acceptable language can be accepted by a DFA whose start state is never visited twice.

Part C. Every DFA-acceptable language can be accepted by a DFA no state of which is ever visited more than once.

Part D. Every DFA-acceptable language can be accepted by a DFA with only a single final state.

Problem 3. *The following problem helps explain our emphasis on looking at languages instead of more general computational problem. It gives a tiny bit of evidence in support of the thesis that studying "the ability to decide" is essentially the same as studying "the ability to compute".*

Suppose you are given an oracle (a "black box" or "magic subroutine") that decides in unit time if a given graph $G = (V, E)$ is three-colorable. (We defined when a graph is *three-colorable* in lecture 1.) Show how to use the algorithm to find a three-coloring of G, when one exists, in $O(|V|)$ time.

Problem 4. Suppose that L is DFA-acceptable. Show that the following languages are DFA acceptable, too.

Part A. $Max(L) = \{x \in L : \text{ there does not exist a } y \in \Sigma^+ \text{ for which } xy \in L\}$.

Part B. $Echo(L) = \{a_1 a_1 a_2 a_2 \cdots a_n a_n \in \Sigma^* : a_1 a_2 \cdots a_n \in L\}$.