Problem Set 8—Due May 27, 2004

Problem 1 *(Counts as 20 points, same as 2 ordinary problems.)*

Part A. Let $L = \{\langle M \rangle : M$ is a TM that accepts some string of prime length$\}$. Prove that L is not decidable.

Part B. Let $L = \{\langle M \rangle : M$ is a TM and $L(M) = L(M)^*\}$. Prove that L is not r.e.

Part C. Let $L = \{\langle M \rangle : M$ is a TM and $L(M) = L(M)^*\}$. Prove that L is not co-r.e.

Part D. Let $L = \{\langle G_1, G_2 \rangle : G_1$ and G_2 are CFGs and $L(G_1) = L(G_2)\}$. Prove that L is not decidable. You may use the fact that $A = \{\langle G \rangle : G$ is a CFG and $L(G) = \Sigma^*\}$ is undecidable.

Problem 2 A *phrase-structure grammar* is like a CFG except that the left-hand side of every rule is a nonempty string of variables: rules look like $\alpha \to \beta$ where $\alpha \in V^+$ and $\beta \in (V \cup \Sigma)^*$. Consider the membership question for phrase-structure grammars: $L = \{\langle G, w \rangle : G$ is a phrase structure grammar and $w \in L(G)\}$. Is L decidable? Is it r.e. or co-r.e.?