Problem Set 1 — Due Tuesday, April 3, 2007

Instructions: Write up your solutions as clearly and succinctly as you can. Typeset solutions, particularly in LaTeX, are always appreciated. Don't forget to acknowledge anyone with whom you discussed problems. Recall that homeworks are due at 3:30 pm sharp on Tuesdays, in the turn-in box in Kemper Hall, room #2131.

Problem 1. Let $G = (V, E)$ be a graph (the "usual" sort, being nonempty, finite, undirected, having no-self loops and no multiple edges). Prove (by giving a convincing argument) or disprove (by giving a smallest counter-example) that the following are equivalence relations for any graph G.

Part A. Let $x, y \in V$. Say that $x\ R_G\ y$ if there is a path in G from x to y (that is, a sequence of vertices $x_1, \ldots, x_n \in V$ $(n \geq 1)$ where each $\{x_i, x_{i+1}\} \in E$ and $x = x_1$ and $y = x_n$).

(Note: the equivalence classes of this equivalence relation are called the "components" of G.)

Part B. Let $x, y \in V$. Say that $x\ R_G\ y$ if x is adjacent to y (that is, $\{x, y\} \in E$).

Part C. Let $x, y \in V$. Say that $x\ R_G\ y$ if $x = y$ or $\{x, y\} \in E$ or there are two vertex-disjoint paths from x to y (paths $x_1, \ldots, x_m$ and $x'_1, \ldots, x'_{n'}$ where $x_1 = x'_1 = x$ and $x_n = x'_{n'} = y$ and $\{x_2, \ldots, x_{n-1} \cap \{x'_2, \ldots, x'_{n'-1} = \emptyset$.

Part D. Let $x, y \in V$. Say that $x\ R_G\ y$ if there is a path from x to y and this remains so even if one removes any edge $e \in E$.

Problem 2. State whether the following propositions are true or false, explaining each answer.

Part A. $\emptyset$ is a language.

Part B. $\emptyset$ is a string.

Part C. ϵ is a language.

Part D. ϵ is a string.

Part E. Every language is infinite or has an infinite complement.

Part F. Some language is infinite and has an infinite complement.

Part G. The set of real numbers is a language.

Part H. There is a language that is a subset of every language.

Part I. The Kleene-star (Kleene closure) of a language is always infinite.

Part J. The concatenation of an infinite language and a finite language is always infinite.

Part K. There is an infinite language L containing the emptystring and such that L^i is a proper subset of L^* for all $i \geq 0$.

Problem 3 Friday is a holiday[1]; don't come to class. Instead, find something to read on the web on César Chavéz. Or join me on Friday from 1:15 pm to 3:15 pm in room 1065 Kemper, when I'll screen the well-regarded documentary *The Fight in the Fields* (1997). But double-check our course web page Thursday night after 11 pm or Friday morning to confirm that I was actually able to get hold of the film.[2]

[1] How annoying is it to have class begin on a Wednesday when the following Friday is a holiday? They couldn't postpone the start of the term until May 2?

[2] Problem 3 will not be graded and the film is an activity unrelated to your work in this particular class.