Problem Set 10 — Due Tuesday, June 5, at 3:30 pm

Problem 1. State whether the following claims are true or false, briefly explaining your answer.

- **a.** $A \le_{\mathrm{m}} A$.
- **b.** If $A \le_{\mathrm{m}} B$ and $B \le_{\mathrm{m}} C$, then $A \le_{\mathrm{m}} C$.
- **c.** If $A \le_{\mathrm{m}} B$ then $\overline{A} \le_{\mathrm{m}} \overline{B}$.
- **d.** If A is r.e. and $A \le_{\mathrm{m}} \overline{A}$ then A is recursive.
- **e.** If A is recursive, then $A \le_{\mathrm{m}} a^*b^*$.
- **f.** If A is r.e., then $A \le_{\mathrm{m}} A_{\mathrm{TM}}$.

Problem 2. Suppose you are given a polynomial time algorithm D that, on input of a Boolean formula ϕ, decides if ϕ is satisfiable. Describe an efficient procedure S that finds a satisfying assignment for ϕ. How many calls to D do you make?

Problem 3. Let $\mathit{MULT\text{-}SAT} = \{\langle\phi\rangle \mid \phi \text{ has at least ten satisfying assignments}\}$. Show that *MULT-SAT* is NP-complete.

Problem 4. A graph $G = (V, E)$ is said to be *k-colorable* if there is a way to paint its vertices using colors in $\{1, 2, \ldots, k\}$ such that no adjacent vertices are painted the same color. When k is a number, by *kCOLOR* we denote the language of (encodings of) k-colorable graphs. The language *3COLOR* is NP-Complete. (You can assume this.) Use this to prove that the language *4COLOR* is NP-Complete, too.

Problem 5. Let

$$D = \{\langle p\rangle : \ p \text{ is a polynomial (in any number of variables) and } p \text{ has an integral root.}\}$$

Prove that D is NP-hard. Is it NP-complete?