Problem Set 5

Problem 1. Decide if the following languages are regular or not. Prove your answers.

Part A. $L = \{w \in \{0,1,2\}^* : w$ has an equal number of 01's and 10's$\}$.

Part B. $L = \{w \in \{0,1\}^* : w$ has an equal number of 01's and 10's$\}$.

Problem 2. Give an algorithm to solve the following decision question: given a regular expression α, is $L(\alpha) = (L(\alpha))^R$?

Problem 3. Are the following statements true or false? Either prove the statement or give a counter-example to it.

Part A. If $L \cup L'$ is regular than L and L' are regular.

Part B. If L^* is regular than L is regular.

Part C. If LL' is regular than L and L' are regular.

Part D. If L and L' agree on all but a finite number of strings, then one is regular iff the other is regular.

Part E. If R is regular, L is not regular, and L and R are disjoint, then $L \cup R$ is not regular.

Problem 4. Define $A = \{x \in \{a, b, \sharp\}^* : x$ contains an equal number of a's and b's or x contains consecutive $\sharp$s or letters$\}$. Prove that A is not regular.

Hint: let $h : \Sigma \to \Gamma^*$ be function and extend h to strings and then to languages in the natural way. Show that if C is regular then so is $h(C)$. We say that "the regular languages are closed under homomorphisms." Consider using this, as well as Problem 3E.

Problem 5. Give a context free grammar for $L = \{a^n b^m : n \neq 2m\}$. Make your grammar unambiguous—and explain why it is unambiguous.

Problem 6. Prove that the context-free languages are closed under reversal.