Problem Set 7

Problem 1.

a. Prove that $L_a = \{a^i b^j c^k : j = \max\{i, k\}\}$ is not context free.

b. Prove that $L_b = \{b_i \# b_{i+1} : b_i \text{ is } i \text{ in binary}, i \geq 1\}$ is not context free.

Problem 2. If A and B are languages define $A \diamondsuit B = \{xy | x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$.

a. Show that if A and B are regular then $A \diamondsuit B$ might not be regular.

b. Show that if A and B are regular then $A \diamondsuit B$ is context free.

Problem 3. Formally specify (draw a transition diagram for) a Turing machine that, when started on an initially empty, two-way infinite tape, will eventually visit any cell. Make your machine be as simple and have as few states as you can.