Problem Set 9 — Due Wednesday, May 30, at 3:30 pm

Problem 1. Prove whether each of the following languages is **RECURSIVE**, **RE** but not recursive, **CO-RE** but not recursive, or **NEITHER** r.e. nor co-r.e.

Part A. $L = \{\langle M, w\rangle : M$ is a TM that uses at most 17 tape squares when run on $w\}$.

Part B. $L = \{\langle M, k\rangle : M$ is a TM that accepts at least one string of length $k\}$.

Part C. $L = \{\langle M, k\rangle : M$ is a TM that diverges on at least one string of length $k\}$.

Part D. $L = \{\langle M, k\rangle : M$ is a TM that accepts a string of length k and diverges on a string of length $k\}$. Assume that the underlying alphabet has at least two characters.

Part E. $L = \{\langle M\rangle :\ M$ is a TM that accepts some palindrome$\}$.

Part F. $L = \{\langle G_1, G_2\rangle :\ G_1$ and G_2 are CFGs which generate the same CFL.$\}$.

Problem 2.

Part A. Give two languages L_1 and L_2, each r.e. but not recursive, with empty intersection.

Part B. Give two languages L_1 and L_2, each r.e. but not recursive, with union Σ^*.

Part C. Are there languages L_1 and L_2 meeting conditions (A) and (B) simultaneously? Why or why not?

Problem 3 Prove that L is decidable iff L is listable in lexicographic order. (A language is listable in lexicographic order if some program outputs $x_1\sharp x_2\sharp x_3\sharp\cdots$, $L = \{x_1, x_2, x_3, \ldots\}$, and $x_1 < x_2 < x_3 < \cdots$ where "$<$" denotes the usual lexicographic ordering on strings.)