Problem Set 3 – Due Tuesday, April 20, 2010, at 4:15 pm

Problem 1. Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA. We say that M accepts a string x in the *all-paths sense* if *every* computation of M on x ends in a state in F; that is, $\hat{\delta}(\{q_0\}, x) \subseteq F$. Let $L'(M)$ denote the set of all $x \in \Sigma^*$ such that M accepts x in the all-paths sense. Show that L is regular iff $L = L'(M)$ for some NFA M.

Problem 2. Do Sipser exercises 1.19(b) and 1.21(b) from page 86.

Problem 3. Find a regular expression representing the encoding of binary numbers divisible by 3. Show your work in systematically devising this regular expression, starting from a DFA for the same language.

Problem 4. Prove that the following languages are not regular.

Part A. $L = \{www : \ w \in \{a, b\}^*\}$.

Part B. $L = \{a^{2^n} : \ n \geq 0\}$.

Problem 5. Decide if the following languages are regular or not, proving your answers either way.

Part A. $L = \{w \in \{0, 1\}^* : \ w$ is *not* a palindrome $\}$.

Part B. $L = \{w \in \{0, 1\}^* : w$ has an equal number of 01's and 10's$\}$.

Part C. $L = \{w \in \{0, 1, 2\}^* : w$ has an equal number of 01's and 10's$\}$.

Problem 6. Let α be a regular expression. Show that there is a regular expression β having the exact same length as α and satisfying $L(\beta) = (L(\alpha))^R$.