The Post Correspondence Problem

As I stated it in class, in the *Post Correspondence Problem* (PCP) (named for Emil Post) you are given sets of words $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_m\}$, each $x_i, y_j \in \Sigma^*$. Here Σ is a fixed alphabet with two or more characters, say $\Sigma = \{a, b\}$. You are asked if you can arrive at the same (nonempty) word z by concatenating either a sequence of words drawn from X or, alternatively, a sequence of words drawn from Y. In other words, are there indices $i_1, \ldots, i_s \in \{1, \ldots, n\}$ and $j_1, \ldots, j_t \in \{1, \ldots, m\}$ (for some s and t) such that $x_{i_1} x_{i_2} \cdots x_{i_s} = y_{j_1} y_{j_2} \cdots y_{j_t}$? The problem is undecidable.

Here is a more restricted (and more traditional) version of the problem. You are given sets of words $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ with each $x_i, y_i \in \Sigma^*$. You are asked if there are indices $i_1, i_2, \ldots, i_s$ (for some s) such that $x_{i_1} x_{i_2} \cdots x_{i_n} = y_{i_1} x_{i_2} \cdots y_{i_n}$. The problem is undecidable.