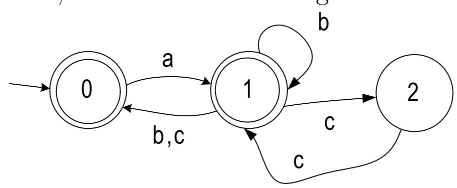
Problem Set 3 – Due Friday, April 19, 2013

Problem 1. For the following problems, do not "simplify" your work (except you should please not indicate unreachable states in any DFA)—show everything.

Using the procedure shown in class, convert the following NFA into a DFA for the same language.



Problem 2. Recall PS2, problem 4. We wanted to prove that the DFA-acceptable languages were closed under homomorphism, but this was awkward to do using the DFA-based definition of the DFA-acceptable languages. Give a *direct* proof that the *regular languages*—the languages of regular expressions—are closed under homomorphisms. When I say to give a direct proof, I mean that you should not employ or even mention DFAs or NFAs: just stick with regular languages / regular expressions.

Problem 3 For any $n \geq 1$, let $L_n = \{0,1\}^*\{1\}\{0,1\}^n$. Prove that there is an NFA for L_n having $n+2$ states. Then prove that there is no DFA for L_n having $2^n - 1$ states. Interpret the *meaning* of this result in plain English.

Problem 4 Let $\text{Dbl}(L) = \{a_1a_1a_2a_2\cdots a_na_n \in \Sigma^* : a_1a_2\cdots a_n \in L\}$. Prove that the DFA-acceptable languages are closed under Dbl.

Problem 5. Consider applying the product construction to NFAs $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ in order to show that the NFA-acceptable languages are closed under intersection.

(a) Formally specify the product machine $M = (Q, \Sigma, \delta, q_0, F)$.

(b) Does the construction work? that is, is $L(M) = L(M_1) \cap L(M_2)$? Prove your result either way.