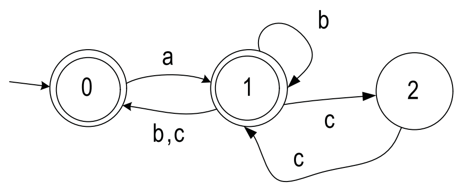
Problem Set 4 – Due Friday, April 26, 2013

Problem 1. Using the procedure shown in class, convert the following NFA into a regular expression for the same language.



Problem 2. Imagine converting an n-state, c-character DFA $M = (Q, \Sigma, \delta, q_0, F)$ into a (fully parenthesized, explicit concatenation symbol) regular expression α for the same language. Upper bound $|\alpha|$ in terms of n and c.

Problem 3. Using the pumping lemma, show that the following languages are not regular.

(a) $L = \{a^{2^n} :\ n \geq 0\}$.

(b) $L = \{www :\ w \in \{a, b\}^*\}$.

(c) $L = \{0^n 1^m 0^n :\ m, n \geq 0\}$.

Problem 4. Let $L = \{w \in \{0, 1\}^* :\ w \text{ is a palindrome}\}$. In class we proved, using the pumping lemma, that L is not regular. Prove the same result using the Myhill-Nerode theorem.

Problem 5. Define $A = \{x \in \{a, b, \sharp\}^* : x$ contains an equal number of a's and b's or x contains consecutive $\sharp$s or consecutive letters$\}$.

(a) Can you use the pumping lemma to prove that A is not regular? Explain.

(b) Prove that A is not regular.

Problem 6. Are the following statements true or false? Either prove the statement or give a simple counter-example.

(a) If $L \cup L'$ is regular then L and L' are regular.

(b) If L^* is regular then L is regular.

(c) If LL' is regular then L and L' are regular.

(d) If L and L' agree on all but a finite number of strings, then one is regular iff the other is regular.

(e) If R is regular, L is not regular, and L and R are disjoint, then $L \cup R$ is not regular.

(f) If L differs from a non-regular language A by a finite number of strings F, then L itself is not regular.