Problem Set 5 – Due Friday, May 3, 2013

Problem 1. Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$, define $\Lambda(M) = \{x \in \Sigma^* : \ \delta^*(q_0, x) \subseteq F\}$. In clear English, explain what is $\Lambda(M)$. Then prove that L is regular iff there is a machine M such that $L = \Lambda(M)$.

Problem 2. Specify a CFG for the language

$$L = \{x \in \{a, b, c\}^* : x \text{ contains an equal number of two different characters}\}.$$

Make your CFG as simple as possible. (If it isn't obviously right to the TA, it isn't right.)

Problem 3. Specify a CFG for $L = \{x \neq y : \ x, y \in \{0,1\}^+ \text{ and } x \neq y\}$.[1] With diagrams or clear English, explain how your grammar works.

Problem 4. Specify a PDA for the language of problem 2.

Problem 5. Consider the following CFG $G = (V, \Sigma, R, \text{STMT})$:

STMT → ASSIGN | IFTHEN | IFTHENELSE
IFTHEN → **if condition then** STMT
IFTHENELSE → **if condition then** STMT **else** STMT
ASSIGN → **a:=1**

with V being the variables in CAPS and Σ being the tokens in **bold**. We explained in class why G (or something just like it) is ambiguous. Provide an unambiguous CFG G', the simplest you can find, where $L(G') = L(G)$. Explain why G' is unambiguous.

Problem 6.

Part A. Prove that every regular language is context free. Do this by by converting a DFA $M = (Q, \Sigma, \Delta, q_0, F)$ into a CFG $G = (V, \Sigma, R, S)$ for the same language.

Part B. Prove that every regular language is generated by an unambiguous CFG.

Part C. Prove that every nonempty CFL is generated by an ambiguous CFG.

[1] $L \subseteq \{0, 1, \neq\}^*$; the first "$\neq$" is the definition of L is just a formal symbol.