Problem Set 6 – Due Friday, May 10, 2013

Problem 1. Using the pumping lemma, prove that $L = \{b_i\#b_{i+1} : b_i \text{ is } i \text{ in binary}, i \geq 1\}$ is not context free.

Problem 2. Alice tries to prove that the language $L = \{1^i + 1^j = 1^{i+j} : \quad i, j \geq 0\}$ is not context free using the pumping lemma.[1] Alice assumes for contradiction that L is context free and lets "p" be the pumping length for L as guaranteed by the pumping lemma. Alice lets s be the string $1^p + 1^p = 1^{2p}$. The string s is in L and has length at least p, so the pumping lemma tells us that s can be partitioned into $uvxyz$ where $|vy| \geq 1$ and $|vxy| \leq p$ and $uv^ixy^iz \in L$ for all $i \geq 0$.

Try to finish Alice's proof. Does any case give you trouble? If so, which? Is it possible to prove that L is not context free by selecting a different string s?

Problem 3 An **unrestricted grammar** $G = (V, \Sigma, R, S)$ is like a CFG except that the rules R are a finite subset of $(V \cup \Sigma)^* V (V \cup \Sigma)^* \times (V \cup \Sigma)^*$. Derivations work just like derivations in a CFG: if there is a rule $\alpha \to \beta \in R$ and you see α within a sentential form, you can replace it by β. The language of G, $L(G)$ is the set of terminal strings derivable from the start symbol S.

Exhibit an unrestricted grammar for the (not-context-free) language $L = \{xx : \; x \in \{a, b\}^*\}$. In English, briefly explain how your grammar works.

[1] Here "+" and "=" are just characters of the alphabet Σ over which strings from L are drawn.