Quiz 1

Neatly print your **name:**

Instructions: *No notes/books/gadgets/neighbors. Be mathematically precise.*

1. An **alphabet** is $\boxed{}$. We call the points in an alphabet *characters*. A **string** is a finite $\boxed{}$.
   Sipser's book defines a **language** as a $\boxed{}$. In class, Prof. Rogaway criticized that definition, saying that one should also add in that $\boxed{}$.

2. Let A and B be languages. Then we define language
   $AB = A \circ B$ as $\boxed{}$.
   The language A^0 is the language $\boxed{}$. For $i \geq 1$, we define A^i recursively, letting A^i be $\boxed{}$. The Kleene closure of A, denoted A^*, is defined as $\boxed{}$.

3. You are given a first DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ with $|Q| = 10$ states, $|F| = 5$ of them final. You are given a second DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ with $|Q| = 10$ states, $|F| = 5$ of them final. Suppose you use the product construction to make a DFA $M = (Q, \Sigma, \delta, s, F)$ for $L(M_1) \cup L(M_2)$. It will have $|Q| = \boxed{}$ states and $|F| = \boxed{}$ of them will be final. *Answers are numbers.*

4. Carefully explain what it **means** if I say: "the DFA-acceptable languages are closed under union." Don't indicate if the statement is true or false—just provide a precise mathematical translation of the meaning of the claim.

5. List, in lexicographic order, the first **five** strings of $\{a, b\}^* - \{a\}^*$. Assume $a < b$.

6. Briefly describe the approach we took towards proving that any DFA for some particular language L needs to have <u>at least</u> n states, for some particular number n.

7. Darken the correct box. No justification is required. If you're not sure, guess.

(a) **True** **False** $\emptyset^*$ is a language.

(b) **True** **False** ε is a language.

(c) **True** **False** Every language is infinite or has an infinite complement.

(d) **True** **False** Some language is infinite and has an infinite complement.

(e) **True** **False** The set of real numbers is a language.

(f) **True** **False** There is a language that is a subset of every language.

(g) **True** **False** The Kleene closure (the star) of a language is always infinite.

(h) **True** **False** The concatenation of an infinite language and a finite language is always infinite.

(i) **True** **False** An algorithm is known to decide if map can be colored with three colors (adjacent regions getting distinct colors).

(j) **True** **False** An algorithm is known to decide if a polynomial $P(x_1, \ldots, x_n)$ with integer coefficients has an integer root.

(k) **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and $F = Q$ then $L(M) = \Sigma^*$.

(l) **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and $F = \emptyset$ then $L(M) = \emptyset$.

(m) **True** **False** If A and B are DFA-acceptable then so is $A \cap B$.

(n) **True** **False** If there's a 10-state DFA that accepts L then there's a 20-state DFA that accepts L.

(o) **True** **False** If L is finite then there is a DFA that accepts L.

(p) **True** **False** It is possible to list distinct bytes $B_1, B_2, \ldots, B_{256}$ in such a way that successive bytes differ at exactly one bit position.

8. Let

$$L = \{0, 11, 110, 1001, 1100, 1111, 10010, \ldots\}$$

be the binary encoding of all nonnegative numbers divisible by 3, no leading 0's allowed. Draw a **DFA** L. Make it as small (=fewest states) as possible.

Hint: ***Step 1:*** *construct a DFA for a language L' like L but where leading 0's and the empty string* ***are*** *allowed.* ***Step 2:*** *add some extra states—three will suffice—to fix the problems.*