Quiz 2

Neatly print: **Firstname LASTNAME:**

Instructions: No notes/books/gadgets/neighbors. Be mathematically precise.

1. You are given a first DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ with $|Q_1| = 10$ states, $|F_1| = 4$ of them final. You are given a second DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ with $|Q_2| = 10$ states, $|F_2| = 4$ of them final. Suppose you use the product construction to make a DFA $M = (Q, \Sigma, \delta, s, F)$ for $L(M_1) \oplus L(M_2)$, the symmetric difference of $L(M_1)$ and $L(M_2)$. It will have $|Q| =$ ☐ states and $|F| =$ ☐ of them will be final. *Answers are numbers.*

2. Recall that, for an NFA $M = (Q, \Sigma, \delta, q_0, F)$, we defined $\delta^*(q, x)$ so as to be the set of all states in Q reachable from q_0 by an x-labeled path. Given this, we could say that M accepts x if ☐ . [mathematically rigorous statement involving δ^* and the components of M.]

3. In stating the Myhill-Nerode theorem, we associated to any regular language $L \subseteq \Sigma^*$ a binary relation $\sim$ by saying that $x \sim x'$ if ☐ . [Make sure to include all needed quantifiers.] As an example, language $L = (0110)^*$ induces a relation $\sim$ where $0 \not\sim 1$, because ☐. The Myhill-Nerode theorem tells us that, if L is a language and $\sim$ is the relation associated to it, then L is regular iff ☐ .

4. There is a 2^{n+1}-state **DFA** $M = (Q, \Sigma, \delta, q_0, F)$ for $L = \{0,1\}^*\{1\}\{0,1\}^n$: namely, let $Q = \{0,1\}^{n+1}$, $q_0 = 0^{n+1}$, $F = \{1\}\{0,1\}^n$, and $\delta(a_0 a_1 \cdots a_n, a) =$ ☐ .

5. Draw the smallest **NFA** you can for the language $L = \{0,1\}^*\{1\}\{0,1\}^3$.

6. State the **pumping lemma** for regular languages. Be careful and explicit with all quantifiers.

7. Darken the correct box. No justification is required. If you're not sure, guess.

   (a) **True** **False** Every language is infinite or has an infinite complement.

   (b) **True** **False** The set of real numbers is a language.

   (c) **True** **False** The Kleene closure (the star) of a language is always infinite.

   (d) **True** **False** $L = \{w \in \{0,1\}^* :\ w$ has an equal number of 01's and 10's$\}$ is regular.

   (e) **True** **False** If $L \subseteq \Sigma^*$ is *not* regular and $h\colon \Sigma \to \Sigma^*$ is a homomorphism, then $h(L)$ is not regular.

   (f) **True** **False** If A and B are regular and h is a homomorphism then $h(A \cup B) = h(A) \cup h(B)$.

   (g) **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is an **NFA** and $F = Q$, then $L(M) = \Sigma^*$.

   (h) **True** **False** An NFA $M = (Q, \Sigma, \delta, q_0, F)$ **rejects** a string x if there is a path from q_0 to a nonfinal state q of M and where the concatenation of the arc-labels along this path is x.

   (i) **True** **False** If α is a regular expression then there is a regular expression β the language of which $\overline{L(\alpha)}$.

   (j) **True** **False** For all $n \geq 0$, $L_n = \{a^i b^i\colon\ i \leq n\}$ is regular.

   (k) **True** **False** If L is regular, the even-length strings of L are regular.

   (l) **True** **False** If the even-length strings L are regular and the odd-length strings of L are regular then L is regular.

8. If $L \subseteq \Sigma^*$ is a language, let $\text{Prefix}(L) = \{x \in \Sigma^*\colon\ xy \in L \text{ for some } y \in \Sigma^*\}$.

   (a) Is $\text{Prefix}(L) \subseteq L$?

   Is $\text{Prefix}(L) \supseteq L$?

   (b) Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$, explain how to construct a DFA $M' = (Q', \Sigma, \delta', q_0', F')$ for $\text{Prefix}(L)$. Explain the construction in a sentence or two of simple English.