Quiz 3

Neatly print: **Firstname LASTNAME:**

Instructions: No notes/books/gadgets/neighbors. Be mathematically precise.

1. You are given a 10-state **NFA** M. Let $L = L(M)$. Create from M a **DFA** M' for $(LL)^*$ using constructions given in class. Then M' will have [ ] states.

2. In stating the Myhill-Nerode theorem, we associated to any language $L \subseteq \Sigma^*$ a binary relation $\sim$ by saying that $x \sim x'$ if [ ]. [Make sure to include all needed quantifiers.] As an example, language $L = \{a^n b^n c^n : \ n \geq 0\}$ induces a relation $\sim$ where $a^5 \not\sim a^6$, because [ ].

3. In class and in Sipser's book, a **PDA** was defined as a six-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where δ has domain [ ] and range [ ]. [Remember that our PDAs are nondeterministic.]

4. Consider the CFG

   $S \to SaSb \mid SbSa \mid \varepsilon$

   Then the parse trees:

   demonstrate that G is **ambiguous**, as $s =$ [ ] is the yield of **both** trees.

5. State the **pumping lemma** for context-free languages. Be careful and explicit with all quantifiers.

6. Darken the **correct** box. No justification is required. If you're not sure, guess.

   (a) **True** **False** The Kleene closure (the star) of a CFL is context free.

   (b) **True** **False** $L = \{ww\colon\ w \in \{0,1\}^*\}$ is context free.

   (c) **True** **False** $L = \{ww^R\colon\ w \in \{0,1\}\}$ is context free.

   (d) **True** **False** $L = \{x\sharp y\colon\ x, y \in \{a,b\}^*$ and $x \neq y\}$ is context free.

   (e) **True** **False** Every regular language can be generated by a CFG $G = (V, \Sigma, R, S)$ in which all rules are of the form $A \to aB$ or $A \to \varepsilon$ (where $A, B \in V$ and $a \in \Sigma$).

   (f) **True** **False** For any CFG G there's a CFG G' such that G' is not ambiguous and $L(G) = L(G')$.

   (g) **True** **False** The CFLs are closed under **symmetric difference** (xor)

   (h) **True** **False** If L is context free, the even-length strings of L are context free.

   (i) **True** **False** If the prime-length strings of L are context free and the composite-length strings of L are context free then L itself is context free.[1]

   (j) **True** **False** No dogs came to "dog day."

7. Classify each of the following languages as: **Reg**: regular; or **CF**: context free but not regular; or **No**: not context free. Then, fully **justify** each answer.

   (a) L is all strings over $\Sigma = \{1, 2, 3, a, b, c\}$ where all numbers in the string precede all letters in the string.

   (b) L is all strings over $\Sigma = \{1, 2, 3, a, b, c\}$ where the number of numbers in the string equals the number of letters in the string.

[1] A number $n \geq 2$ is *prime* if $ab = n$ implies $a = 1$ or $b = 1$. Here $a, b \in \{1, 2, 3, \ldots\}$ are natural numbers. A number $n \geq 4$ is *composite* if it is not prime.