Midterm Exam — Spring 2014

First name: ________ **LAST NAME:** ________

Seat #: ________ **Student ID #:** ________

Instructions:

• The exam is closed-book, closed-notes, closed-devices, closed-neighbors. Anyone violating these rules will be fed to the saltwater crocodiles that live in the basement of Kemper.
• Write neatly. If the person grading can't easily read what you wrote, it's wrong.
• Where answers depend on conventions, use those of Sipser or the lectures.
• An "efficient" algorithm: one that runs in *polynomial*, not *exponential*, time.
• **Good luck!**

On page	you got	out of
1		50
2		45
3		45
4		60
Σ		200

Problem Z: For this question you should specify a **range** (ie, a [lowerbound, upperbound] pair) for which you have **98% confidence**—not more and not less—that the correct answer lies within the range you specify.

The average distance between the **earth** to the **moon** is between

________ and ________ **times** the **circumference of the earth**.
(lowerbound) (upperbound)

1 Fill in the blank

*Fill in the boxes. All answers are **numbers**.*

1. A **DFA** $M = (Q, \Sigma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 5$ characters. Then there are $\boxed{}$ points in the domain of δ and $\boxed{}$ points in the range of δ.

An **NFA** $M = (Q, \Sigma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 5$ characters. Then there are $\boxed{}$ points in the domain of δ and $\boxed{}$ points in the range of δ.

A **PDA** $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 2$ characters in the input alphabet and $|\Gamma| = 4$ characters in the tape alphabet. Then there are $\boxed{}$ points in the domain of δ and $\boxed{}$ points in the range of δ.

2. You are given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ with $|Q| = 100$ states, operating on $|\Sigma| = 2$ characters, and employing $|F| = 10$ final states.

If we construct from M an NFA for $(L(M))^R$ it will have $\boxed{}$ states.

Alternatively, if we construct from M an NFA for $\overline{L(M)}$ by combining procedures seen in class, it will have $\boxed{}$ states.

3. A CNF (Chomsky-Normal Form) CFG $G = (V, \Sigma, R, S)$ has 7 variables, 6 terminals, 5 rules of the form $A \to BC$, and 8 rules of the form $A \to a$. Using the procedure shown in class and in your book, convert G to a PDA M that allows one to push an arbitrary number of symbols onto the stack in one transition. Then M will have $|Q| = \boxed{}$ **states**. If, instead, we allow M to push only one symbol onto the stack at a time (which was our original convention), then the construction will need to have $|Q| = \boxed{}$ **states**.

4. As per PS5, problem 5, there's a natural conversion of a DFA $M = (Q, \Sigma, \delta, q_0, F)$ into a context-free grammar $G = (V, \Sigma, R, S)$ for $L(M)$. Suppose that M has $|Q| = 10$ states, $|\Sigma| = 3$ characters, and $|F| = 5$ final states. Then the corresponding CFG G will have $|V| = \boxed{}$ variables and $|R| = \boxed{}$ rules.

2 Short answer

1. Use the **pumping lemma** (and anything else that you might need) to prove that $L = \{x \in \{a,b\}^* :\ x$ is **not** a palindrome$\}$ is **not** regular.

2. In the Myhill-Nerode theorem an equivalence relation $\sim$ is associated to any language L. We did this by defining, for a given L, that $x \sim x'$ if *(complete the sentence)*

Let $L = \{x \in \{a,b\}^* :\ x$ **is** a palindrome$\}$. Then $x = aa$ and $x' = aaa$ **are** / **aren't** (please circle the **correct** choice) $\sim$ related to one another because *(complete the sentence)*

3. Give a **CFG** for the language $L = \{xx^R yy^R :\ x, y \in \{a,b\}^*\}$. Your CFG must be as **simple as possible**.

4. Draw a **picture** that helps explain the main idea behind the **pumping lemma** for **CFLs**. You picture should include labels S, A, and u, v, x, y, z. Then, with no more than a couple of sentences, **explain the proof idea** that your picture aims to convey.

5. Carefully explain what it **means** if I say: "the CFLs are closed under intersection. Don't indicate if the statement is true or false—just provide a precise mathematical translation of the meaning of the claim.

Now **prove** or **disprove** (circle one) the claim: **the CFLs are closed under intersection.**

6. Let L be the set of all decimal digits d such that d occurs **infinitely often** in the decimal representation of the number $\pi = 3.14159\cdots$.

Is L **regular**? Circle either: **yes** or **no**.

Now **prove your answer**.

3 True or False

Darken (completely fill in) the **correct** answer. If you don't know an answer, please **guess**.

1. **True** **False** The concatenation of finite languages A and B is finite.
2. **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and $F \neq \emptyset$ then $L(M) \neq \emptyset$.
3. **True** **False** Every regular language can be accepted by an NFA with only one final state.
4. **True** **False** If L^* is infinite the L is infinite.
5. **True** **False** Regular expressions are strings.
6. **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA then $x \in L(M)$ iff $\delta^*(q_0, x) \cap F \neq \emptyset$.
7. **True** **False** The pumping lemma can be used to show that languages are regular.
8. **True** **False** Let $L = \{a^n b^n : n \geq 1\}$. Then L^* is regular.
9. **True** **False** If $A \subseteq L \subseteq B$ and A and B are regular then L is regular.
10. **True** **False** If $L \cup L'$ is regular then L and L' are regular.
11. **True** **False** If $L \oplus L'$ is finite then L regular iff L' is regular.
12. **True** **False** The image $h(L)$ of a context-free language L under a homomorphism h is context free.
13. **True** **False** $L_\pi = \{\texttt{3}, \texttt{31}, \texttt{314}, \texttt{3141}, \texttt{31415}, \texttt{314159}, \ldots\}$ is regular.
14. **True** **False** Complementing the final state set of an NFA M gives an NFA for $\overline{L(M)}$.
15. **True** **False** For a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we defined δ^* as $\delta = \bigcup_{i \geq 0} \delta^i$.
16. **True** **False** $L = \{w \in \{0,1\}^* : w$ contains an equal number of 01's and 10's$\}$ is regular.
17. **True** **False** The intersection of a CFL and a regular language is context free.
18. **True** **False** An efficient algorithm is known to decide if two NFAs accept the same language.
19. **True** **False** An efficient algorithm is known to decide if $w \in L(\alpha)$ for a regular expression α.
20. **True** **False** An efficient algorithm is known to decide if $w \in L(G)$ for a CNF CFG G.