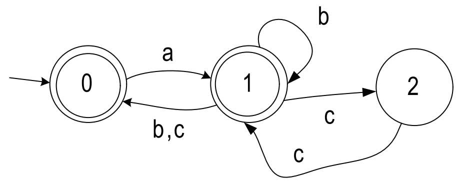
Problem Set 4 – Due Friday, April 25, 2013

Problem 1.

(a) Using the procedure shown in class, convert NFA into a regular expression for the same language.



(b) Using the procedure shown in class, convert the regular expression $(ab^* \cup c)^*$ into an NFA for the same language.

(c) Suppose that a (fully parenthesized) regular expression α over the alphabet Σ has
c characters from Σ,
o composition symbols,
s stars, and
u union symbols. Convert α it to a DFA M for the same language using the procedures seen in class. How many states will M have?

Problem 2. Without consulting sources other than your book, provide a well-written proof of the following "strong" form of the pumping lemma:

> If L is regular then there exists a number p such that for all $S = s_1\, s\, s_2 \in L$, $|s| \geq p$, there are strings x, y, z, $s = xyz$, $1 \leq |y| \leq p$, such that $s_1\, xy^iz\, s_2 \in L$ for all $i \geq 0$.

Problem 3. Prove that the following languages are not regular.

(a) $L = \{x \in \{a,b\}^* :\ x \text{ is not a palindrome}\}$.

(b) $L = \{w{=}w:\ w \in \{0,1\}^*\}$. (The second $=$ is a character from the alphabet $\{0,1,=\}$ that L is over.)

(c) $L = \{a^{2^n}:\ n \geq 0\}$.

Problem 4. Let $L = \{xx^R:\ x \in \{a,b\}^+\}$. Use the Myhill-Nerode theorem to prove that L is not regular.

Problem 5. Define $A = \{x \in \{a,b,\sharp\}^* : x$ contains an equal number of a's and b's or x contains consecutive $\sharp$s or consecutive letters$\}$.

(a) Can you use the pumping lemma to prove that A is not regular? Explain.

(b) Prove that A is not regular.

Problem 6. Are the following statements true or false? Either prove the statement or give a simple counter-example.

(a) If $L \cup L'$ is regular then L and L' are regular.

(b) If L^* is regular then L is regular.

(c) If LL' is regular then L and L' are regular.

(d) If L and L' agree on all but a finite number of strings, then one is regular iff the other is regular.

(e) If R is regular, L is not regular, and L and R are disjoint, then $L \cup R$ is not regular.

(f) If L differs from a non-regular language A by a finite number of strings F, then L itself is not regular.