Problem Set 5 – Due Friday, May 2, 2014

Problem 1. Are the following languages regular? Prove your answers. You don't need to know anything about the number $\pi \in \mathbb{R}$ except that it's irrational (meaning: it's not the ratio of two integers).

(a) L_a = the set of all decimal digits d such that d occurs infinitely often in the decimal representation of the number $\pi = 3.14159\cdots$.

(b) $L_b = \{3, 31, 3141, 31415, 314159, \ldots\}$, the set of all nonempty prefixes of the decimal expansion of π, ignoring the decimal point.

Problem 2. In an well-written essay of about a page (200-300 words), discuss the following question. *We have defined various computational models in our class. Some aspects of each definition were* ***essential****—the definition has to look basically that way to capture the desired notion—while other aspects were* ***inessential****—maybe as insignificant as arbitrary conventions just needed to make things concrete. Select a computational model covered in class (DFAs, NFAs, or regular expressions) and explore definitional alternatives that* ***do*** *and that* ***do not*** *change the essence of the object defined.*

Problem 3. Specify a CFG for the language

$$L = \{x \in \{a, b, c\}^* : x \text{ contains an equal number of two different characters}\}.$$

Make your CFG as simple as possible. (If it isn't obviously right to the TA, it isn't right.)

Problem 4. Specify a CFG for $L = \{x \neq y : \ x, y \in \{0,1\}^+ \text{ and } x \neq y\}$.[1] With diagrams or clear English, explain how your grammar works.

Problem 5. Prove that every regular language is context free. Do this by by converting a DFA $M = (Q, \Sigma, \delta, q_0, F)$ into a CFG $G = (V, \Sigma, R, S)$ for the same language.

[1] $L \subseteq \{0, 1, \neq\}^*$; the first "$\neq$" is the definition of L is just a formal symbol.