Problem Set 6 – Due Friday, May 9, 2014

Problem 1. Let $L = \{w \in \{a, b, c\}^* : \ w$ has an equal number of a's, b's, and c's$\}$. Describe a PDA for $\overline{L}$ in English, and then formally specify it.

Problem 2. Let $h\colon \Sigma_\varepsilon \to \Gamma^*$ be an arbitrary function. Extend it to strings and then languages by way of $h(\varepsilon) = \varepsilon$, $h(a_1 a_2 \cdots a_n) = h(a_1)h(a_2)\cdots h(a_n)$ and then $h(L) = \{h(x)\colon x \in L\}$. Here $a_1, \ldots, a_n \in \Sigma$ and $L \subseteq \Sigma^*$. The function h, and its extension to strings and languages, is called a *homomorphism*. Are the CFLs closed under homomorphism? Prove your answer.

Problem 3.

a. Prove that $L_a = \{a^i b^j c^k : j = \max\{i, k\}\}$ is not context free.

b. Prove that $L_b = \{b_i \# b_{i+1} : b_i$ is i in binary, $i \geq 1\}$ is not context free.

Problem 4. Are the following languages context free? Prove your answers either way.

a. $L = \{ww^R : w \in \{a, b\}^*\}$

b. $L = \{ww^R w : w \in \{a, b\}^*\}$

Problem 5. Answer true or false, proving each answer.

(a) Every regular language is generated by an unambiguous CFG.

(b) Every regular language is generated by an ambiguous CFG.

(c) The CFLs are closed under intersection.

(d) Every subset of a regular language is context free.

(e) If L is context free then $L \cap a^*b^*a^*$ is context free.

Problem 6 A queue is similar to a stack, except that pushing and popping happen at opposite ends: symbols are pushed onto the *top* of the queue and popped, and read, from the *bottom* of the queue. A queue automaton (QA) is like a PDA but has a queue instead of a stack. Your formalization of a QA can be either deterministic or nondeterministic. Describe a language that is recognized by a QA but not by any PDA.