Problem Set 8 – Due Friday, May 23, 2014

If you liked working with a partner and want to do so again, you may, turning in one problem set per group. I don't recommend groups of more than two, but if you have a good partnership going with three, I won't complain.

Problem 1 Recall our friend the queue automaton (QA): it was just like a PDA but writes symbols onto the front of the queue and reads symbols off the back of the queue. Carefully explain why QAs can do all and only what TMs can do.)

Problem 2. Guess the classification for each of the following languages as either **recursive**, **r.e.** but not not co-r.e., **co-r.e.** but not r.e., or **neither** r.e. nor co-r.e. (For ease of grading, please use the bold-face label.) Briefly justify when something *is* decidable, Turing-acceptable, or has a Turing-acceptable complement.

A $\{\langle M\rangle : M$ is a TM that accepts some string of prime length$\}$.

B $\{\langle M, k\rangle : M$ is a TM that accepts at least one string of length $k\}$.

C $\{\langle M, k\rangle : M$is a TM that accepts at most one string of length $k\}$.

D $\{\langle M\rangle : M$ is a TM and M has 100 states$\}$.

E $\{\langle M\rangle : M$ is a TM and $L(M) = L(M)^*\}$.

F $\{\langle M\rangle : M$ is a TM and $L(M) = \emptyset\}$.

G $\{\langle M\rangle : M$ is a TM and $L(M)$ is r.e.$\}$.

H $\{\langle M\rangle : M$ is a TM and $L(M)$ is decidable$\}$.

I $\{\langle M, k\rangle : M$is a TM that runs forever on at least one string of length $k\}$.

J $\{\langle M\rangle : M$ is a C-program that halts on $\langle M\rangle\}$.

K $\{\langle M, k\rangle :$ M is a TM that accepts a string of length k and diverges on a string of length $k\}$. Assume that the underlying alphabet has at least two characters.

L $\{\langle M, w\rangle : M$ is a TM and M that uses at most 20 tape cells when run on $w\}$.

M $\{\langle G\rangle : G$ is a CFG and G accepts an odd-length string$\}$.

N $\{\langle G_1, G_2\rangle : G_1$ and G_2 are CFGs and $L(G_1) = L(G_2)\}$.

O $\{\langle M\rangle :$ M is a smallest (fewest-state) NFA for $L(M)\}$.

Problem 3 Say that a language $L = \{x_1, x_2, \ldots\}$ is *enumerable* if there exists a two-tape TM M that outputs $x_1\sharp x_2\sharp x_3\sharp\cdots$ on a designated *output tape*. The other tape is a designated *work tape*, and the output tape is write-only, with the head moving only from left-to-right. Say that L is *enumerable in lexicographic order* if L is enumerable, as above, and, additionally, $x_1 < x_2 < x_3 < \cdots$, where "$<$" denotes the usual lexicographic ordering on strings.

A. Prove that L is r.e. iff L is enumerable. (This explains the name "recursively enumerable.")

B. Prove that L is recursive iff it is enumerable in lexicographic order.