# Problem Set 9 – Due Friday, May 30, 2014

**Problem 1.** As you did last week, classify each of the following languages as **recursive**, **r.e.** but not decidable, **co-r.e.** but not decidable, or **neither** r.e. nor co-r.e. Giving reductions where appropriate, prove your results. *This problem will count as 30 points, triple a conventional problem.*

**A** $A = \{\langle M, k\rangle : M$ is a TM that accepts at least one string of length $k\}$.

**B** $B = \{\langle M, k\rangle : M$is a TM that runs forever on at least one string of length $k\}$.

**C** $C = \{\langle M, k\rangle :$ $M$ is a TM that accepts a string of length $k$ and diverges on a string of length $k\}$. Assume that the underlying alphabet has at least two characters.

**D** $D = \{\langle M\rangle :$ $M$ is a TM that accepts some palindrome$\}$.

**E** $E = \{\langle G_1, G_2\rangle :$ $G_1$ and $G_2$ are CFGs and $L(G_1) \oplus L(G_2) = \emptyset\}$.
You may assume that $L = \{\langle G\rangle :$ $G$ is a CFG and $L(G) = \Sigma^*\}$ is undecidable.

**F** $F = \{\langle M\rangle :$ $M$ is a TM and $L(M)$ is recursive$\}$.

**Problem 2** Prove or disprove each of the following claims.

**A.** $A \leq_{\mathrm{m}} A$.

**B.** If $A \leq_{\mathrm{m}} B$ and $B \leq_{\mathrm{m}} C$, then $A \leq_{\mathrm{m}} C$.

**C.** If $A \leq_{\mathrm{m}} B$ then $\overline{A} \leq_{\mathrm{m}} \overline{B}$.

**D.** If $A$ is r.e. and $A \leq_{\mathrm{m}} \overline{A}$ then $A$ is recursive.

**E.** If $A$ is recursive, then $A \leq_{\mathrm{m}} a^*b^*$.

**F.** If $A \leq_{\mathrm{m}} B$ then $B \leq_{\mathrm{m}} A$.

**G.** If $A \leq_{\mathrm{m}} B$ and $B \leq_{\mathrm{m}} A$ then $A = B$.