Quiz 2

First name: ☐ **Last name:** ☐ **Seat #:** ☐

Instructions: No notes/books/gadgets/neighbors. Be mathematically precise.

1. Let's recall the **product construction**, as used to show that the DFA-acceptable languages are closed under **union**. Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA for L_1 and let $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA for L_2. We construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ for $L_1 \cup L_2$ by asserting that $Q =$ ☐ , that $\delta((p, q), a) =$ ☐ , that $q_0 =$ ☐ , and, finally, that $F =$ ☐ . If $|Q_1| = N_1$, $|F_1| = n_1$, $|Q_2| = N_2$, and $|F_2| = n_2$, then $|F| =$ ☐ *(formula involving N_1, n_1, N_2, n_2).*

2. Suppose we use the **subset construction** to convert a 7-state NFA $M = (Q, \Sigma, \delta, q_0, F)$ into a DFA $M' = (Q', \Sigma, \delta', q_0, F')$ for the same language. Then M' will have $|Q'| =$ ☐ states *(a number).* If $|F| = 2$ then M' will have $|F'| =$ ☐ final states *(a number).*

3. The string `00100` is the ☐ th binary string in lexicographic order (assume `0` < `1`).

4. **Darken** the **correct** box. No justification is required. If you're not sure, guess.

   (a) **True** **False** Every finite language is regular.

   (b) **True** **False** If A and B are regular then $A - B$ is regular.

   (c) **True** **False** If L^* is infinite the L is infinite.

   (d) **True** **False** Regular expressions are strings.

   (e) **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA then $x \in L(M)$ iff $\delta^*(q_0, x) \subseteq F$.

   (f) **True** **False** If NFA $M = (Q, \Sigma, \delta, s, F)$ has $F = Q$ then $L(M) = \Sigma^*$.

   (g) **True** **False** The pumping lemma can be used to show that languages are regular.

   (h) **True** **False** Let $L = \{a^n b^n : n \geq 1\}$. Then L^* is regular.

   (i) **True** **False** An n-state NFA M can be converted into an n^2-state DFA for $L(M)$.

   (j) **True** **False** A regular expression α can be converted into a $2|\alpha|$-state NFA for $L(\alpha)$.

   (k) **True** **False** If $A \subseteq L \subseteq B$ and A and B are regular then L is regular.

   (l) **True** **False** In the Myhill-Nerode theorem, we defined $x \sim_L x'$ by: $[x \in L \Leftrightarrow x' \in L]$.

5. Carefully state the **pumping lemma** for regular languages. Use any form of this lemma you like, but make sure it's nontrivial and all quantifiers are clear.

6. Draw an **NFA** for $L = \{aab, aba\}$. Your NFA must use **as few states and arrows** as possible.