Midterm

Instructions:

- Relax, breathe, be calm. It ain't so bad.
- Make sure to indicate your seat number—both letter and number—on your exam.
- Please do not sit next to a friend. That means anyone with whom you've ever discussed course material and whose name you know (first name, last name, or nickname). *Next to* means the person immediately to your left, right, front, front-left, or front-right.
- Cell phones off. No notes/books/gadgets.

First name: ______ **Last name:** ______ **Seat #:** ______

I will not and did not cheat on this exam: ______

Your signature

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1. Prof. Rogaway criticized Sipser's definition of a *language* as a *set of strings*; he said that one should add in that [ ].

2. Let $L = \{b, ab\}$. List the first six strings of L^* in lexicographic order (assume $a < b$):
[ ] [ ] [ ] [ ] [ ] [ ]

3. A **DFA** is a five-tuple $M = (Q, \Sigma, \delta, q_0, F)$ where Q is a finite set, Σ is an alphabet, $q_0 \in Q$, $F \subseteq Q$, and δ is a function with domain [ ] and range [ ]. In contrast, the transition function for an **NFA** has domain [ ] and range [ ].

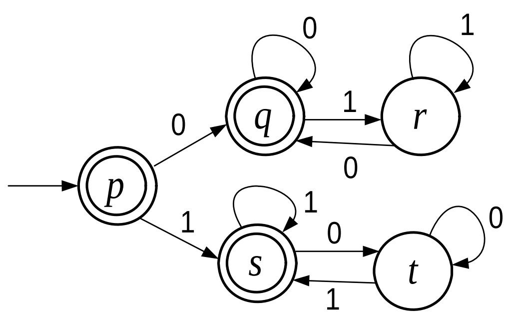
4. We showed in class that the DFA-acceptable languages were closed under intersection. The construction that established this is called the [ ]. It works liked this. Given a DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and a DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ we construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ for $L(M_1) \cap L(M_2)$. For the state set of M we select $Q =$ [ ]. (Thus if M_1 has 10 states and M_2 has 20 states, then M will have [ ] states.) For the start state of M we set $q_0 =$ [ ]. The final states of M are set $F =$ [ ].

5. Suppose you use the procedure illustrated in class to convert a 100-state NFA M with ε-transitions into an NFA M' without any ε-transitions. How many states will M' have? [ ].

6. Complete the following narrative about the **Myhill-Nerode** theorem. Let $L \subseteq \Sigma^*$ be a language. Define an associated equivalence relation $\sim \subseteq \Sigma^* \times \Sigma^*$ by declaring that $x \sim x'$ iff [ ] . Define the *block* (or *equivalence class*) containing a string x as $[x] = \{$ $\}$. As a concrete example, if $L = (111)^*$ then $[1] = \{$ $\}$. (←Be concrete in naming the elements; don't use $\sim$). The Myhill-Nerode theorem says that L is regular iff [ ] .

7. Suppose we use the procedure illustrated in class to convert the regular expression $(a \cup b^*)$ into an NFA M. How many states will M have? [ ] . (Count carefully; no partial credit.)

8. Consider the following DFA M. Describe the language $B \subseteq \{0, 1\}^*$ it accepts in a simple and clear sentence.



9. Suppose you wish to show that there's no *smaller* (fewer-state) DFA for the language B of problem 9. One approach is to show that the DFA M has **no pair of equivalent states** (for the equivalence relation of homework 2, problem 5). For example, $q \not\sim s$ because [ ] .

10. Suppose you you wish to show that there's no *smaller* (fewer-state) DFA than M for the language B of problem 9. One approach is to use the **pigeonhole principle** and a proof by contradiction. Give the first few sentences of such a proof. You do not need to go through the full case analysis; doing any one case is fine.

11. In PS #2 we defined $\mathcal{E}(L) = \{x \in L : \text{there exists a } y \in \Sigma^+ \text{ for which } xy \in L\}$.

    Let $L = ((0 \cup 1)(0 \cup 1))^*$. Then $\mathcal{E}(L) =$ [ ] . (Write it in the simplest form (fewest characters) you can.

12. Carefully state the **pumping lemma** for regular languages. Be careful to make quantification clear.

13. Use the pumping lemma (and any other tools you need) to show that
    $L = \{x \in \{a,b\}^* : x \text{ is not a palindrome}\}$
    is not regular.

14. A context-free grammar (CFG) is a triple $G = (V, \Sigma, R, S)$ where V (the *variables*) is a finite set, Σ (the *terminals*) is an alphabet, S (the *start symbol*) is an element of V, and R (the *rules*) is [ ] .
    (Answer with a mathematical description of the kind of "thing" that R is.)

15. **Darken** the **correct** box. No justification is needed. If you're not sure, guess (grading is based on the number of correct answers).

(a) **True** **False** If L_1 and L_2 are regular then $L_1 \cap L_2$ is regular.

(b) **True** **False** If $L_1 \cap L_2$ is regular then L_1 and L_2 are regular.

(c) **True** **False** If L^* is regular then L is regular.

(d) **True** **False** If LL' is regular then L and L' are regular.

(e) **True** **False** If L and L' coincide on all but a finite number of strings, then one is regular iff the other is regular.

(f) **True** **False** There's a language L where $L = L^R$ but L contains no palindrome.

(g) **True** **False** There's a language L_0 that's a subset of every language.

(h) **True** **False** If L is accepted by an NFA then $\overline{L}$ is accepted by a DFA.

(i) **True** **False** If $|L| = 1$ then L^* is infinite.

(j) **True** **False** The concatenation of finite languages A and B is finite.

(k) **True** **False** Every subset of a regular language is regular.

(l) **True** **False** There's an algorithm to determine if a multivariate polynomial has an integer root, but nobody knows its running time.

(m) **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $F = Q$ then $L(M) = \Sigma^*$.

(n) **True** **False** If L is an language then $L^* - L^+ = \{\varepsilon\}$.

(o) **True** **False** If $G = (V, \Sigma, R, S)$ is a context free grammar (CFG) all of whose rules are of the form $A \to B$ or $A \to Bc$ (where $A, B \in \Sigma$ and $c \in \Sigma$), then $L(G) = \emptyset$.

(p) **True** **False** Let $G = (V, \Sigma, R, S)$ and let $w \in L(G)$. Then G either has one parse tree with yield w or else it has none at all.