## Problem Set 3 – Due Friday, April 17, 2015

**Problem 1.** Using the procedure shown in class, convert the following NFA into a DFA for the same language. Show all work.



**Problem 2.** Using the procedure shown in class, eliminate all  $\varepsilon$ -arrows from the following NFA.



- **Problem 3.** Let  $L_1, L_2, L_3 \subseteq \Sigma^*$  be languages and let  $Most(L_1, L_2, L_3)$  be the set of all  $x \in \Sigma^*$  that are in at least two of  $L_1, L_2, L_3$ . Prove: if  $L_1, L_2$ , and  $L_3$  are DFA-acceptable then so is  $Most(L_1, L_2, L_3)$ .
- **Problem 4** Let  $Stutter(L) = \{a_1a_1 \ a_2a_2 \ \cdots \ a_na_n \in \Sigma^* : a_1a_2 \ \cdots \ a_n \in L\}$ . (A) Prove that the DFA-acceptable languages are closed under *Stutter*. (B) Then, having proved it once, give another, entirely different proof.

**Problem 5.** How many states are in the smallest possible DFA for  $\{0,1\}^* \{1^{10}\}$ ? Prove your result.

**Problem 6** Let  $L_n$  (for  $n \ge 1$ ) be  $\{0,1\}^*\{1\}\{0,1\}^n$ . Prove that there is an NFA for  $L_n$  having n+2 states, but that there is no DFA for  $L_n$  having  $2^n - 1$  or fewer states. In a well written English sentence or two, give a high-level interpretation of your result.