ECS 120 Final — Spring 2014

First name: 　　　　**LAST NAME:**

Seat row: 　　**Seat #:** 　　**Student ID #:**

Signature:

I solemnly swear that I am the above student, and that the work of this exam is entirely my own.

Hints for success:

Fill out everything on this page except for the scores. Read the questions carefully; they might not ask what you expect. If you don't understand what a question means, please ask. Make your writing legible, logical, and succinct. Write in grammatical English. Be mathematically rigorous. If a definition is requested, use the definition given in class or your text, not something equivalent. Some abbreviations:

$A_{\text{TM}} = \{\langle M, w\rangle : \text{ TM } M \text{ accepts } w\}$
CFG = context free grammar
CFL = context free language
CNF = Chomsky normal form
DFA = deterministic finite automaton
NFA = nondeterministic finite automaton
NTM = nondeterministic Turing machine
PDA = pushdown automaton
TM = Turing machine
r.e. = recursively enumerable = Turing Acceptable
co-r.e. = the class of languages with r.e. complements
rec = recursive = decidable

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4		30
5		30
6		30
Σ		175

Who is **Kim Kardashian**?

Who is **Edward Snowden**?

Who is **Stephen Hawking**?

Yes, you really do need to answer the three questions above (or say "I don't know").

1 True or False

Indicate if the following statements are true or false, by **filling in** (darkening) the correct box. Do not provide any justification. If in doubt, **guess**; missing answers will be treated as wrong.

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1. **True** **False** Every set is a language.
2. **True** **False** The union of infinitely many regular languages is decidable.
3. **True** **False** If L is regular then so is $\{xx\colon\ x \in L\}$.
4. **True** **False** If L is regular then so is $\{xy\colon\ x, y \in L\}$.
5. **True** **False** The Myhill-Nerode Theorem can be used to show a language not regular.
6. **True** **False** Let $A = \{1^{2^i}\colon\ i \geq 0\}$. Then A^* is regular.
7. **True** **False** $\{w \in \{0,1\}^*\colon\ w$ has an equal number of 01's and 10's$\}$ is regular.
8. **True** **False** For every number n, the language $L_n = \{0^i 1^i :\ i \leq n\}$ is regular.
9. **True** **False** If some 10-state NFA accepts L then some 100-state DFA accepts L.
10. **True** **False** If $L_1 \cup L_2$ is regular then L_1 and L_2 are regular.
11. **True** **False** If A is regular and B is context free then $A \cap B$ is context free.
12. **True** **False** Every subset of a context free language is regular.
13. **True** **False** If L is context free then L^* is context free.
14. **True** **False** If a CFG G is in CNF, then G is *not* ambiguous.
15. **True** **False** If A and B are regular then $\{xy \mid x \in A$ and $y \in B$ and $|x| = |y|\}$ is context free.
16. **True** **False** There exists an algorithm to decide if a regular expression α is a shortest regular expression for $L(\alpha)$.
17. **True** **False** There's a polynomial time algorithm to decide if two DFAs accept the same language.
18. **True** **False** The pumping lemma for CFLs is useful for showing languages context free.
19. **True** **False** The rules (productions) of a CFG $G = (V, \Sigma, R, S)$ are an *arbitrary* subset of the infinite set $V \times (V \cup \Sigma)^*$.
20. **True** **False** Given $h\colon \Sigma \to \{0,1\}^*$, define $h(L) = \{h(a_1) \cdots h(a_n)\colon\ a_1 \cdots a_n \in L\}$. Then L is context free if and only if $h(L)$ is context free.

21. **True** **False** The intersection of CFLs is a CFL.

22. **True** **False** There's a polynomial time algorithm to decide if a string w is in the language of a CNF CFG G.

23. **True** **False** There's a polynomial time algorithm to decide if two CFGs generate the same language.

24. **True** **False** If L^* is recursive then L is recursive.

25. **True** **False** A language L is either r.e. or co-r.e..

26. **True** **False** The decidable languages are closed under complement.

27. **True** **False** The r.e. languages are closed under complement.

28. **True** **False** Exploiting the power of guessing, nondeterministic Turing machines can decide language A_{TM}.

29. **True** **False** A Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\mathrm{accept}}, q_{\mathrm{reject}})$, as defined in class, can output a uniformly random bit $b \in \{0, 1\}$ and then halt.

30. **True** **False** A Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\mathrm{accept}}, q_{\mathrm{reject}})$, as defined in class, can be designed such that it visits every tape cell infinitely often.

31. **True** **False** If $A \leq_{\mathrm{m}} B$ and $B \leq_{\mathrm{P}} C$ then $A \leq_{\mathrm{m}} C$.

32. **True** **False** If $A \leq_{\mathrm{m}} B$ and A is decidable then B is decidable.

33. **True** **False** If A is decidable then $A \leq_{\mathrm{m}} \{1\}$.

34. **True** **False** If $L_1 \in \mathbf{P}$ and $L_2 \in \mathbf{P}$ then $L_1 - L_2 \in \mathbf{P}$.

35. **True** **False** A polynomial-time algorithm is known to decide if a graph G has a clique of size k.

36. **True** **False** Every language in **NP** is decidable.

37. **True** **False** If $L_1 \in \mathbf{NP}$ and $L_2 \in \mathbf{NP}$ then $L_1 \cup L_2 \in \mathbf{NP}$.

38. **True** **False** NP stands for *non-polynomial time*; it contains all the decidable languages outside of **P**.

39. **True** **False** Suppose $A \leq_{\mathrm{P}} B$ and $A \in P$. Then $B \in \mathbf{P}$.

40. **True** **False** If there's a polynomial time algorithm to decide if a boolean formula ϕ is satisfiable then there's a polynomial time algorithm to decide if a graph G has a 3-coloring.

2 Language Classification

Classify as:
- **recursive** recursive (decidable)
- **r.e.** r.e. (Turing-acceptable) but not decidable
- **co-r.e.** co-r.e. (Turing-acceptable complement) not decidable
- **neither** neither r.e. nor co-r.e.

No explanation is required. If in doubt, **guess**; missing answers will be treated as wrong.

1. $\{\langle M \rangle : M$ is a TM and $L(M)$ is regular$\}$

2. $\{\langle G \rangle : G = (V, \Sigma, R, S)$ is a CFG and $L(G) = \emptyset\}$

3. $\{\langle G_1, G_2 \rangle : G_1$ and G_2 are CFGs and $L(G_1) = L(G_2)\}$

4. $\{\langle G \rangle : G$ is a CFG and $L(G)$ is infinite$\}$.

5. $\{\langle M \rangle : M$ is an NFA and $L(M) = \{0, 1\}^*\}$

6. $\{\langle M, w \rangle : M$ is a TM and M rejects $w\}$

7. An undecidable language L for which $L \leq_{\mathrm{m}} A_{\mathrm{TM}}$.

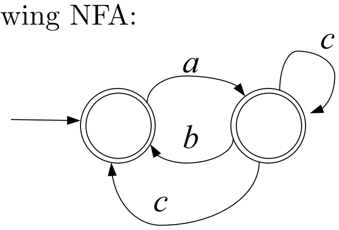
8. $\{d$: the digit $d \in \{0, 1, \ldots, 9\}$ appears infinitely often in the decimal representation of the real number $\pi\}$.

9. SAT $= \{\phi : \phi$ is a satisfiable Boolean Formula$\}$

10. $\{p : p$ is a polynomial (with integer coefficients, in any number of variables) and p has an integral root.$\}$.

3 Short Answers

1. List, in order ($a < b < c$), the lexicographically first four strings in the language of the following NFA:



2. State the **Cook-Levin Theorem**.

3. A **PDA** $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 2$ characters in the input alphabet and $|\Gamma| = 5$ characters in the tape alphabet. Then there are ☐ points in the domain of δ. (Answer is a number.)

4. Let $L = \{a, ab\}$. Let $\sim_L$ be the equivalence relation associated to L by the Myhill-Nerode theorem. Then $\sim_L$ has ☐ equivalence classes. (Answer is a number.) (No justification is required or will be looked at, but here is some space if you need it.)

5. If $G = (V, \Sigma, R, S)$ is a CNF (Chomsky Normal Form) CFG and `hello` $\in L(G)$, how many steps (rule applications) will it take to derive x from S? ☐ (Answer is a number.)

6. Complete the following definition: A context-free language L is **inherently ambiguous** if . . . *(Do not use the word "ambiguous" in your definition.)*

7. Complete the following definition:

   Given languages A and B, we say that $A \leq_{\mathrm{P}} B$ if:

8. *Complete the following definition*:

   A language L is in **NP-complete** if:

9. Let $L = \{\langle M \rangle : M$ is a TM and $L(M)$ contains a palindrome$\}$. (Recall that a string x is a palindrome if it reads the same forward and backward: $x = x^R$.) Use an NTM to explain why L is r.e..

10. When we define the language $A_{\mathrm{TM}} = \{\langle M, w \rangle :$ TM M accepts $w\}$, what is the purpose of the **angle brackets** (the $\langle\ \rangle$ symbols) that surround M, w?

11. Let $F = \{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is finite}\}$. Prove that $A_{\text{TM}} \leq_{\text{m}} F$. I'll start you out.

We want to map $\langle M, w \rangle$ to $\langle M' \rangle$ by a Turing-computable function f in such a way that
M accepts $w \Rightarrow L(M')$ is finite and
M doesn't accept $w \Rightarrow L(M')$ is infinite.
To accomplish this, we will have M', on input $x \in \Sigma^*$, behave as follows:

Now we need to show that the construction works. To do so, we observe that:

- *If M accepts w then $L(M')$ is* $\boxed{}$ *and*
- *If M doesn't accept w then $L(M')$ is* $\boxed{}$

where N is the number of steps it takes M to accept w. *(Don't say "finite" or "infinite"; be specific.)*

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12. A graph $G = (V, E)$ is *k-colorable* if there is a way to paint its vertices using colors in $\{1, 2, 3, \ldots, k\}$ such that no adjacent vertices are painted the same color. Let kCOLOR be $\{\langle G \rangle : G \text{ is } k\text{-colorable}\}$. The language 3COLOR is NP-Complete. (You can assume this.) Use this to prove that 4COLOR is NP-Complete. I'll start you out:

- *First we must show the "easy" part, that* $\boxed{}$. *This is true because one can guess the four coloring of $G = (V, E)$ and then verify it's validity.*

- *Now for the reduction: we must show* $\boxed{} \leq_{\text{P}} \boxed{}$. *To show this, map $G = (V, E)$ to $G' = (V', E') = f(G)$ by saying that*

$V' = \boxed{}$ *and*

$E' = \boxed{}$.

To show this reduction correct, observe that:

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