Problem Set 1 — Due Friday, April 9, 1999

Instructions: Write up your solutions as clearly and succinctly as you can. Don't forget to acknowledge anyone with whom you discussed problems. Recall that homework is due at 4:10 pm in the box in Eng. II, room #2131.

Problem 1. Give a precise and concise name for each of the following sets. (See Chapter 0 for notation.)

a. The smallest alphabet Σ such that $L = \{1, 3, 5, 7, 9, 11, 13, \ldots\}$ is a language over Σ.

b. $\{n \mid n = 2m \text{ for some } m \in \mathcal{N}\}$.

c. $\{n \mid n \text{ is an integer and } n = n + 1\}$.

d. $\{n \mid n \text{ is an integer and } n = 2n\}$.

e. $\{1^{2^n} : n \in \mathcal{N}\}$. *Note: this is an infinite language. We are using a^b in two different senses.*

f. $\bigcup_{i=0}^{10} \{0,1\}^i$.

g. $\{w : w \text{ is a binary string and there is a string } x \text{ which is a substring of both } w \text{ and } w^R\}$.

Problem 2. Show that at a party of 20 people, there are at least 2 people who have the same number of friends present at the party. Assume (however unrealistically) that friendship is symmetric and anti-reflexive. *Hint*: Carefully use the pigeonhole principle.

Problem 3. Give DFAs for the following languages. Assume $\Sigma = \{0, 1\}$.

a. The set of all strings with 010 as a substring.

b. The set of all strings which do not have 010 as a substring.

c. The set of all strings which have an even number of 0's or an even number of 1's.

d. The complement of $(1 \cup 10)^*$.

e. The binary encodings of numbers divisible by 3: $0^*\{\varepsilon, 11, 110, 1001, 1100, 1111, \ldots\}$.

Problem 4 State whether the following proposition are true or false. Prove each answer.

a. Every DFA-acceptable language can be accepted by a DFA with an even number of states.

b. Every DFA-acceptable language can be accepted by a DFA whose start state is never visited twice.

c. Every DFA-acceptable language can be accepted by a DFA no state of which is ever visited more than once.

d. Every DFA-acceptable language can be accepted by a DFA with only a single final state.