Problem Set 9 — Due Wednesday, June 2, 1999

Problem 1. [20 points] Classify the following languages as **decidable**, **r.e.** (but not co-r.e.), **co-r.e.** (but not r.e.), or **neither** r.e. nor co-r.e.. Prove all your answers, giving decision procedures or reductions.

A. $L = \{\langle M \rangle : M$ accepts some even-length string$\}$.

B. $L = \{\langle M, w \rangle : M$ is a TM which uses at most 17 tape squares when run on $w\}$

C. $L = \{\langle M \rangle :\ M$ accepts some palindrome$\}$.

D. $L = \{\langle M \rangle :\ M$ never prints a "0" (regardless of the input)$\}$.

E. $L = \{\langle \phi \rangle :\ \phi$ is a Boolean formula which has *no* satisfying assignment$\}$.

F. $L = \{\langle G_1, G_2 \rangle :\ G_1$ and G_2 are CFGs which generate the same CFL.$\}$.

G. $L = \{\langle \alpha \rangle :\ \alpha$ is shortest regular expression for $L(\alpha)\}$.

Problem 2. [10 points] Finish the proof of Rice's theorem by arguing the case when the emptyset does have property P.

Problem 3. [10 points] Suppose you are given a subroutine which, on input of a Boolean formula ϕ, decides if ϕ is satisfiable. (See your text, Chapter 7, for a definition of thw satisfiablity problem.) Describe an efficient procedure which finds a satisfying assignment for ϕ. Show that it is adequate to call the subroutine $n - \lg n$ times to obtain the satisfying assignement for an n-variable formula.