Midterm — Section 1

Instructions: Please answer the questions succinctly and thoughtfully. Good luck.

— Phil Rogaway

Name:

Signature:

On problem	you got	out of
1		45
2		30
3		25
Σ		100

1 Short Answer [45 points]

(1) Let M_1 be an n_1-state DFA and let M_2 be an n_2-state DFA. Using the procedures given in class and in your text, how many states will be in the NFA M the language of which is $L(M_1) \cup (L(M_2))^*$? Do not make your machine unnecessarily large. Explain your reasoning.

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(2) Using the procedure we have seen in class, convert the regular expression $1^* \cup 00$ into an NFA for the same language.

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(3) Complete the following, mathematically precise, definition, according to the conventions of our text:

an **NFA** is a 5-tuple $M = ($ $)$ where:

(4) Recall that, for $L \subseteq \{0,1\}^*$, $\text{PAL}(L) = \{x \in \{0,1\}^* : xx^R \in L\}$. Write a regular expression for $\text{PAL}(\{\varepsilon, 00, 01, 000, 0110\}$.

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(5) Carefully describe a decision procedure (algorithm) to decide the following language: $L = \{\langle\alpha\rangle : \alpha$ is a regular expression and the smallest NFA for $L(\alpha)$ is smaller than the smallest DFA for $L(\alpha)\}$. (smallest = fewest states)

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(6) Write a CFG for the language $L = \{1^i \neq 1^j : i, j \geq 1, i \neq j\}$. Use no more than 6 rules.

(7) Use the pumping lemma for regular languages to prove that the following language is **not** regular: $L = \{1^i < 1^j : 1 \leq i < j\}$.

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(8) Prove or disprove: the following grammar is **ambiguous**:

$S \rightarrow S1S1S2S \mid S1S2S1S \mid S2S1S1S \mid \varepsilon$

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(9) Carefully **state** the pumping lemma for context-free languages.

2 Justified True or False [30 points]

Put an **X** through the **correct** box. Where it says "Explain" provide a **brief** (but convincing) justification. No credit will be given to correct answers that lack a proper justification. Where appropriate, **make your justification a counter-example**. Throughout, we use L to denote a language (maybe regular, maybe not).

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1. Every CFL has some ambiguous grammar. **True** **False**

Explain:

2. Every subset of a regular language is regular. **True** **False**

Explain:

3. For every $n \geq 1$, there is an $2n$-state DFA for the language $0\{0,1\}^n$.

Explain: **True** **False**

4. The intersection of two context-free languages is context free. **True** **False**

Explain:

5. In class we exhibited an algorithm to decide if two context-free grammars generate the same language. **True** **False**

6. If L is context free then some NPDA accepts the complement of L.

True **False**

3 A Little Proof [25 points]

Find the smallest DFA for the language $L = (000)^* \cup (111)^*$. Prove that there is no DFA smaller than yours. (smaller=fewer states.)