Problem Set 1 — Due Thursday, January 10, 2002

Instructions: Write up your solutions as clearly and succinctly as you can. Don't forget to acknowledge anyone with whom you discussed problems. Recall that homework is due at 10:00 am in the box in Eng. II, room #0086.

Problem 1. The following question is to remind you about **inductive definitions** and their use.

A. Give an inductive definition for a *decimal number*. A decimal number is a string over the alphabet $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots, \mathbf{9}\}$. Examples are **4**, **120**, **007**.

B. Give an inductive definition for the *value* of a decimal number. This is a map ν from decimal numbers to nonnegative integers.

Problem 2. For each of the following, give an example language L to prove existence, or explain why no such language exists. Assume an underlying alphabet of $\{0, 1\}$.

A. An infinite language with an infinite complement.

B. A language closed under concatenation and containing no even-length strings.

C. An infinite unary language L such that if $x \in L$ and $y \in L$ then there is no string in L of length $|x|+|y|$. (A unary language means that the underlying alphabet has just one character, say **1**.)

D. A finite language having a longest string x that is longer than a longest string of any other finite language.