Midterm Exam

Instructions: This is a closed book, closed notes exam. Do all **4** problems. Do your best to communicate your ideas clearly and succinctly. Good luck. —Phil Rogaway

Name:

On problem	you got	out of
1		40
2		40
3		20
4		20
Σ		120

1 Short Answer [40 points]

1.1 Imagine converting the regular expression $\alpha = (00 \cup 001)^*$ into a DFA M using the constructions given in class and in your text. How many states will M have?

1.2 Now draw the *smallest* DFA that accepts the language $(00 \cup 001)^*$. *(By "smallest" I mean that your machine has the minimum possible number of states.)*

1.3 Give a careful, mathematically precise statement of the ***pumping lemma*** for regular languages.

1.4 Complete the definition: A context free grammar is a 4-tuple $G = (\quad , \quad , \quad , \quad)$, where

1.5 Give a CFG for $L = \{a^i b^j c^k :\ i \neq j \text{ or } j \neq k\}$.

1.6 Prove that the language $L = \{a^i b^j c^k :\ i \neq j \text{ or } j \neq k\}$ is not regular.

1.7 Describe an algorithm that, given regular expressions α_1 and α_2, these denoting distinct regular languages L_1 and L_2, finds a shortest string w that is in one of these languages and not in the other.

1.8 Give the first four strings, in lexicographic order, of $(abc \cup bc)^*$

2 Justified True or False [40 points]

Put an **X** through the **correct** box. Then provide a brief justification. **Where appropriate, make the justification a counter-example.**

2.1 If $L \subseteq A$ and A is regular, then L is regular. **True** **False**

Justification:

2.2 If L^* is regular then L is regular. **True** **False**

Justification:

2.3 If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $Q = F$ then $L(M) = \Sigma^*$.

Justification: **True** **False**

2.4 The complement of a regular language is context free. **True** **False**

Justification:

2.5 Suppose L is an infinite regular language. Then there exists positive numbers a and b such that L contains some string of length $ai + b$ for every $i \geq 0$.

Justification: **True** **False**

2.6 There is a language L for which $L = L^*$. **True** **False**

Justification:

2.7 Every nonempty regular language L is generated by some ambiguous CFG.

Justification: **True** **False**

2.8 There is an infinite language L such that whenever x and y are in L, then there is no string in L having length $|x| + |y|$. **True** **False**

Justification:

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3 Minimal-Size DFA **[20 points]**

Let $L = \{\varepsilon, 0, 1, 00, \ldots, 1111\}$ be the set of all binary strings of length 4 or less. Show that there is no 4-state DFA that accepts L.

4 A Closure Property of Regular Languages [20 points]

For $L \subseteq \{0,1\}^*$ a language, let $\mathcal{S}(L) = \{y \in \{0,1\}^* : \ xy \in L \text{ for some } x \in \{\mathtt{0},\mathtt{1}\}^*\}$.

4.1 Name the strings of $S(L)$ for $L = \{011, 0101\}$:

$$\mathcal{S}(\{011, 0101\}) = \{ \qquad\qquad\qquad\qquad\qquad\qquad \}.$$

4.2 Prove that if L is regular then $\mathcal{S}(L)$ is regular.

Now that wasn't so bad, was it?