Problem Set 2 — Due Tuesday, January 17, 2006

Problem 1. Show that at a party of 10 people, there are at least 2 people who have the same number of friends present at the party. Assume (however unrealistically) that friendship is symmetric and anti-reflexive. *Hint*: Carefully use the pigeonhole principle.

Problem 2 Give DFA for the following languages. Assume $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

Part A. The set of all strings that contain an even number of 0's and at most two 1's. (Sipser, 1.4.l)

Part B. The complement of $(0 \cup 11)^*$.

Part C. The binary encodings of numbers divisible by 5: $0^*\{\varepsilon, 101, 1010, 1111, 10100, 11001, \cdots\}$.

Problem 3

Part A. Show that there is a deterministic finite automaton with $n+1$ states that recognizes the language $(1^n)^*$. (The alphabet is $\Sigma = \{0, 1\}$.)

Part B. Show that there does *not* exist a smaller deterministic finite automaton for this language. (smaller = fewer states).

Problem 4 State whether the following proposition are true or false, proving each answer.

Part A. Every DFA-acceptable language can be accepted by a DFA with an even number of states.

Part B. Every DFA-acceptable language can be accepted by a DFA whose start state is never visited twice.

Part C. Every DFA-acceptable language can be accepted by a DFA no state of which is ever visited more than once.

Part D. Every DFA-acceptable language can be accepted by a DFA with only a single final state.

Problem 5.

Part A. Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$ define an associated binary relation $\approx$ on Σ^* by saying that $x \approx y$ iff $\delta^*(q_0, xz) = \delta^*(q_0, yz)$ for all $z \in \{\mathtt{0}, \mathtt{1}\}^*$. Prove that $\approx$ is an equivalence relation.

Part B. Given a DFA-acceptable language L define an associated binary relation $\equiv$ on Σ^* by saying that $x \equiv y$ iff $(\forall z \in \{\mathtt{0}, \mathtt{1}\}^*)$ $(xz \in L \Leftrightarrow yz \in L)$. Prove that $\equiv$ is an equivalence relation.

Part C. Prove that if L is DFA-acceptable then the associated equivalence relation $\equiv$ has a finite number of equivalence classes.