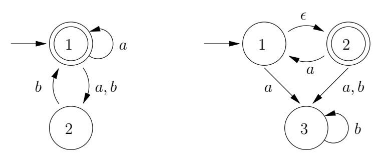
Problem Set 3 — Due Tuesday, January 24, 2006

Problem 1. Give minimal-size NFA (size = # of states) for the following languages. Assume $\Sigma = \{\texttt{0}, \texttt{1}\}$.

1. $\{w : w$ contains an even number of $0s$, or exactly two $1s.\}$
2. The language $0^*1^*0^*0$.

Problem 2. Use the construction given in class to convert the following two nondeterministic automata to equivalent deterministic finite automata.



Problem 3. Suppose that L is DFA-acceptable. Show that the following languages are DFA acceptable, too.

Part A. Let $\Sigma^+ = \{x_1 x_2 \cdots x_k : k \geq 1$ and each $x_i \in \Sigma\}$.
$Max(L) = \{x \in L :$ there does not exist a $y \in \Sigma^+$ for which $xy \in L\}$.

Part B. $Echo(L) = \{a_1 a_1 a_2 a_2 \cdots a_n a_n \in \Sigma^* : a_1 a_2 \cdots a_n \in L\}$.

Part C. $\text{Comb}_{\text{even}}(L) = \{a_2 a_4 a_6 \cdots a_n \in \Sigma^* : a_1 a_2 a_3 \cdots a_n \in L\}$, for n is even.

Problem 3.

(a) Show that there is an $(n+2)$-state NFA for $L_n = (\Sigma^*)0\Sigma^n$. (Take $\Sigma = \{0, 1\}$.)

(b) Prove that *any* DFA for L_n requires at least 2^n states.

Problem 4. Let L be a language over Σ and define the language $\text{PAL}(L) = \{x \in \Sigma^* : xx^R \in L\}$. If L is DFA-acceptable, is $\text{PAL}(L)$ necessarily DFA-acceptable? Prove your answer.