Problem Set 5 — Due Tuesday, February 7, 2006

Problem 1. Exhibit decision procedures (algorithms) which answer the following questions.

(a) Given a DFA M, does $L(M)$ contain a string of the form $(babb \cup aa^*b)^*$?

(b) Given NFAs M_1 and M_2, is $|L(M_1)| = |L(M_2)| < \infty$?

(c) Given a regular expression α, is there a shorter regular expression which denotes the same language?

Problem 2. Use closure properties and the fact that $\{a^n b^n : n \geq 0\}$ is not regular to show that the following languages are not regular.

(a) $L_a = \{a^i b^j c^k : i = j \text{ or } j = k\}$.

(b) $L_b = \{w \in \{a, b\}^* : w \text{ has more } a\text{'s than } b\text{'s}\}$.

Problem 3. Are the following propositions true or false? Give proofs or counterexamples.

(a) If $L_1 \cup L_2$ is regular and L_1 is finite, then L_2 is regular.

(b) If $L_1 \cup L_2$ is regular and L_1 is regular, then L_2 is regular.

(c) If $L_1 L_2$ is regular and L_1 is finite, then L_2 is regular.

(d) If $L_1 L_2$ is regular and L_1 is regular, then L_2 is regular.

(e) If L^* is regular then L is regular.

Problem 4. Exhibit a context free grammar for the language

$$L = \{x_1 \neq x_2 : x_1, x_2 \in \{a, b\}^* \text{ and } x_1 \text{ is not equal to } x_2\}.$$

"$\neq$" is a formal symbol: "$ab \neq bb$" $\in L$, but "$ab \neq ab$" $\notin L$.

Then describe a PDA for the same language. You do not need to be formal about the PDA; explaining how it behaves in English is fine.

Problem 5. Consider the grammar G defined by $S \to AA$, $A \to AAA \mid bA \mid Ab \mid a$.

(a) Carefully and precisely describe the $L(G)$ in an easy-to-recognize form.

(b) Is $L(G)$ regular? Prove your answer either way.

(c) Is G ambiguous? Prove your answer either way.

(d) Is $L(G)$ inherently ambiguous? Give a convincing argument either way.