

## Problem Set 6 — Due Tuesday, February 14, 2006

**Problem 1.** In class, for the language  $\{w \in \{1, 2\}^* : w \text{ contains twice as many 1s as 2s}\}$ , a student suggested the following grammar  $G$ :

$$S \rightarrow 1S1S2 \mid 1S2S1 \mid 2S1S1 \mid SS \mid \epsilon$$

Is  $L = L(G)$ ? Prove your answer either way.

**Problem 2.** An *unrestricted grammar* is like a CFG except the LHS of a rule can be any string from  $(V \cup \Sigma)^* \cup (U \cup \Sigma)^*$ . For example,  $aBc \rightarrow aaCd$  could be in  $R$ .

Give an unrestricted grammar for  $L = \{a^n b^n c^n : n \geq 0\}$ .

**Problem 3.** Show that following languages are *not* context free.

**Part A.**  $L = \{a^i b^j c^k : j = \max\{i, k\}\}$

**Part B.**  $L = \{b_i \# b_{i+1} : b_i \text{ is } i \text{ in binary, } i \geq 1\}$  is not context free.