Problem Set 9 — Due Tuesday, March 7, 2006

Problem 1. Let A and B be two disjoint languages. The recursive language $\mathcal{A}$ *recursively separates* A and B if $A \subseteq \mathcal{A}$ and $B \cap \mathcal{A} = \emptyset$.

A. Show that any two disjoint co-r.e. languages are separable by some decidable language.

B. Show that $A = \{\langle M\rangle : M(\langle M\rangle) = \text{Yes}\}$ and $B = \{\langle M\rangle : M(\langle M\rangle) = \text{No}\}$ are not recursively separable.

Problem 2.

A. Show that

$$L_A = \{\langle M, k\rangle : M \text{ is a TM which accepts at least one string of length } k\}$$

is not decidable.

B. Prove that

$$L_B = \{\langle M, k\rangle : M \text{ is a TM that loops on at least one string of length } k\}$$

is not decidable.

C. Let

$$\begin{aligned} L_C \;=\; & \{\langle M, k\rangle : M \text{ is a TM which accepts some string of length } k, \\ & \text{but } M \text{ loops on some (other) string of length } k\}. \end{aligned}$$

Show that L_C is not acceptable. (Assume the underlying alphabet has at least two characters.)

D. Show that L_C is not co-acceptable, either.

Problem 3. *Counts as two problems.* Classify the following languages as **decidable**, **acceptable** (but not decidable), **co-acceptable** (but not decidable), or **neither** acceptable nor co-acceptable. Prove all your answers, giving decision procedures or reductions.

A. $L = \{\langle M\rangle : M \text{ accepts some even-length string}\}$.

B. $L = \{\langle M, w\rangle : M \text{ is a TM which uses at most 17 tape squares when run on } w\}$

C. $L = \{\langle M\rangle : M \text{ accepts some palindrome}\}$.

D. $L = \{\langle M\rangle : M \text{ never prints a "0" (regardless of the input)}\}$.

E. $L = \{\langle \phi\rangle : \phi \text{ is a Boolean formula which has } \textit{no} \text{ satisfying assignment}\}$.

F. $L = \{\langle G_1, G_2\rangle : G_1 \text{ and } G_2 \text{ are CFGs which generate the same CFL.}\}$. *You may use the fact that* $\{\langle G\rangle : G \textit{ is a CFG and } L(G) = \Sigma^*\}$ *is undecidable.*

G. $L = \{\langle \alpha\rangle : \alpha \text{ is shortest regular expression for } L(\alpha)\}$.