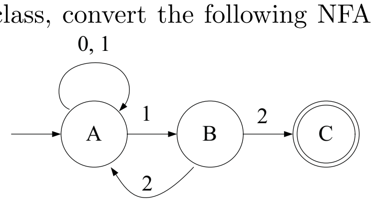
Problem Set 2 – Due Tuesday, January 24, 2012

For any of the following problems, should you need to assume the existence of a non-regular language A, you may do so.

Problem 1. For the following problems, do not "simplify" your work (except you should please not indicate unreachable states in any DFA)—show everything.

(a) Using the procedure shown in class, convert the following NFA into a DFA for the same language.



(b) Using the procedure shown in class, convert the same NFA into a regular expression for the same language.

(c) Using the procedure shown in class, convert the following regular expression into an NFA for the same language:

$$(0 \cup 01)^* \cup 10$$

Problem 2. Let $\mathcal{F}(L) = \{x \in L :$ there exists a $y \in \Sigma^+$ for which $xy \in L\}$.

(a) What is $\mathcal{F}(\{\mathtt{0},\mathtt{1}\}^*)$? What is $\mathcal{F}(\{\varepsilon, 0, 1, 00, 01, 111, 1110, 1111\})$?

(b) Prove that if L is DFA-acceptable then $\mathcal{F}(L)$ is too.

A *prefix* of a string y is a string x such that $y = xx'$ for some x'. A prefix is *proper* if it is not the empty string. For any language L, let $\mathcal{G}(L) = \{w \in L|$ no proper prefix of w is in $L\}$.

(c) What is $\mathcal{G}(\{\mathtt{0},\mathtt{1}\}^*)$? What is $\mathcal{G}(\{\varepsilon, 00, 01, 110, 0100, 0110, 1110, 1111\})$?

(d) Prove that if L is DFA-acceptable then so is $\mathcal{G}(L)$.

Problem 3. Let $L_n = \{1^i : 0 \le i < n\}$ (recall that $1^0 = \varepsilon$). Prove that there is a DFA M having n accepting states that accepts L. Then prove that L cannot be accepted by any DFA having fewer accepting states.

Problem 4. Consider applying the product construction to NFAs $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ in order to show that the NFA-acceptable languages are closed under intersection.

(a) Formally specify the product machine $M = (Q, \Sigma, \delta, q_0, F)$.

(b) Does the construction work? that is, is $L(M) = L(M_1) \cap L(M_2)$? Prove your result either way.

Problem 5. Let $h : \Sigma \to \Sigma^*$ and let $L \subseteq \Sigma^*$ be a language. Let $h(L) = \{h(a_1) \cdots h(a_n) : a_1 \cdots a_n \in L\}$.

(a) Is it true that if L is regular, then so must be $h(L)$? Prove your answer.

(b) Is it true that if $h(L)$ is regular, then so must be L? Prove your answer.