Problem Set 5 – Due Tuesday, February 14, 2011

Problem 1. Consider the following CFG $G = (V, \Sigma, R, \text{STMT})$:

> STMT $\to$ ASSIGN | IFTHEN | IFTHENELSE
> IFTHEN $\to$ **if condition then** STMT
> IFTHENELSE $\to$ **if condition then** STMT **else** STMT
> ASSIGN $\to$ **a:=1**

with V being the variables in CAPS and Σ being the tokens in **bold**. Show that G is ambiguous. Then provide an unambiguous CFG G', the simplest you can find, where $L(G') = L(G)$. Explain why G' is unambiguous.

Problem 2. Formally specify both **(a)** a CFG and **(b)** a PDA for the language

$$L = \{x \in \{a, b, c\}^* : x \text{ contains an equal number of two different characters}\}.$$

Make your CFG and PDA as simple as possible. (If they ain't obviously right, they ain't right!)

Problem 3.

a. Prove that $L_a = \{a^i b^j c^k : j = \max\{i, k\}\}$ is not context free.

b. Prove that $L_b = \{b_i \# b_{i+1} : b_i \text{ is } i \text{ in binary}, i \geq 1\}$ is not context free.

Problem 4. Are the following languages context free? Prove your answers either way.

a. $L = \{ww^R : w \in \{a, b\}^*$

b. $L = \{ww^R w : w \in \{a, b\}^*$

Problem 5. If A and B are languages define $A \diamondsuit B = \{xy | x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$.

a. Show that if A and B are regular then $A \diamondsuit B$ might not be regular.

b. Show that if A and B are regular then $A \diamondsuit B$ is context free. You do not have to write out a CFG or PDA in full; an English language description will suffice.