The Pumping Lemma

Definition: Language L *pumps* if there exists a N such that for all $s \in L$, $|s| \geq N$, s can be decomposed into $s = xyz$, $|y| \geq 1$, such that $xy^iz \in L$ for all $i \geq 0$.

Theorem (The Pumping Lemma): If L is regular, then L pumps.

You use the pumping lemma to prove that a language is *not* regular—you show that the language does not pump. The pumping lemma is useless for showing that a language is regular.

To prove that L is not regular using the pumping lemma, you suppose for contradiction that L were regular. You then let N be the "N" guaranteed by the pumping lemma. Then you choose a $s \in L$ of length at least N, and you show that no matter how you were to try to write $s = xyz$ (subject only to $|y| \geq 1$), there is some $i \geq 0$ for which $xy^iz \notin L$.

If you understand the proof of the lemma, you can remember it and use it correctly. Don't get the "alternating quantifiers" mixed up; it is very important what you get to choose and what you do not get to choose. Also, if you understand the proof, it is easy to see the the following "strong" form of the lemma is true, too:

Theorem ("Strong" pumping lemma): If L is regular, then there exists a N such that for any $s_1ss_2 \in L$ with $|s| \geq N$, s can be decomposed into $s = xyz$, $|y| \geq 1$, such that $s_1xy^izs_2 \in L$ for all $i \geq 0$.

Informally, when using the pumping lemma you do not control exactly how the string you select will be decomposed into xyz; nonetheless, you can imagine a "window" in the string of size N (placed anywhere you like), and you can be guaranteed that the y will appear somewhere within this window.