Problem Set 1 — Due Tuesday, January 19, 1999

Instructions: Write up your solutions as clearly and succinctly as you can. Don't forget to acknowledge anyone with whom you discussed problems. Recall that homework is due at 4:00 pm in the box in Eng. II, room #2131.

Problem 1. We say that a string x over an alphabet consisting of left parenthesis, '(', and right parenthesis, ')' is *1-balanced* if

a. x has an equal number of left and right parentheses; and
b. any suffix of x has at least as many right parentheses as left parentheses.

Say that a string x (over the same alphabet) is *2-balanced* if it can be generated by the following rules:

a. The empty string, ϵ, is 2-balanced.
b. If x is 2-balanced, so is (x).
c. If x and y are 2-balanced, so is xy.
d. Nothing else is 2-balanced.

Part A. Prove that if a string x is 1-balanced, then it is 2-balanced. *Hint*: By induction on $|x|$.

Part B. Prove that if a string x is 2-balanced, then it is 1-balanced. *Hint*: By induction on the definition of 2-balanced.

Problem 2. Give DFAs for the following languages. Assume $\Sigma = \{0, 1\}$.

(a) The set of all strings with 010 as a substring.
(b) The set of all strings which do not have 010 as a substring.
(c) The set of all strings which have an even number of 0's or an even number of 1's.
(d) The complement of $(1 \cup 10)^*$.
(e) The binary encodings of numbers divisible by 3: $0^*\{\varepsilon, 11, 110, 1001, 1100, 1111, \ldots\}$.

Problem 3

(a) Show that there is a deterministic finite automaton with $n+1$ states that recognizes the language $(1^n)^*$. (The alphabet is $\Sigma = \{0, 1\}$.)
(b) Sow that there does *not* exist a smaller deterministic finite automaton for this language. (smaller = fewer states).

Problem 4 State whether the following proposition are true or false, proving each answer.

Part A. Every DFA-acceptable language can be accepted by a DFA with an even number of states.

Part B. Every DFA-acceptable language can be accepted by a DFA whose start state is never visited twice.

Part C. Every DFA-acceptable language can be accepted by a DFA no state of which is ever visited more than once.

Part D. Every DFA-acceptable language can be accepted by a DFA with only a single final state.