Problem Set 7—Due Friday, November 14

Problem 26. Text Problem 24-1 (page 511). *For this problem assume the edge weights are distinct. Define a second best MST as follows: Let T_1 be the MST of G. Then a spanning tree T_2 of G is a* ***second best MST*** *of G if $T_2 \neq T_1$ and for any spanning trees S other than T_1 we have that $w(S) \geq w(T_2)$.*

Problem 27. Given some graph $G = (V, E)$ whose vertices are uniquely labeled 1 through $n = |V|$, we wish to determine the smallest integer k such that deleting vertices 1 through k results in a graph whose largest component has at most $n/2$ vertices. (When we delete a vertex, we also delete all edges incident to that vertex.) Give the most efficient algorithm you can to find k. State the running time of your algorithm and justify its correctness. (*Hint: Use union-find and build the graph backwards.*)

Problem 28. Before you is an infinite 2-dimensional array of 2-bit counters, all initialized to 0. A basic operation is INC(i, j) which increments the counter at row i and column j of the array. Each time a counter is incremented, the cost is one. However, if a counter overflows (i.e. is incremented from 3 to 4), it again becomes 0 and propagates a carry to its neighbors to the North, South, East, and West: array positions $(i-1, j)$, $(i+1, j)$, $(i, j-1)$, and $(i, j+1)$ get incremented. Note that propagated carries may cause other counters to overflow and produce further propagated carries—maybe even back to counter (i, j) itself!

(a) Using the accounting method, show that the amortized cost of INC(i, j) is constant.

(b) Define a potential function for this data structure. Then show the amortized cost of an INC is constant by computing the actual cost plus the change in potential.

Problem 29. Text Problem 18-2 (page 376).