Problem Set 9—Due Friday, December 5

Problem 35 Exercise 36.5–2 (p. 960).

Problem 36 Exercise 36.5–4 (p. 960).

Problem 37. A graph $G = (V, E)$ is said to be k-*colorable* if there is a way to paint its vertices using colors in $\{1, 2, \ldots, k\}$ such that no adjacent vertices are painted the same color. When k is a number, by kCOLOR we denote the language of (encodings of) k-colorable graphs. The language 3COLOR is NP-Complete. (You can assume this.) Use this to prove that the language 4COLOR is NP-Complete, too.

Problem 38. Suppose that someone gives you a polynomial-time algorithm for deciding LP: on input of a weighted graph G and a number B, the algorithm says if G has a (simple) path of weight $\geq B$. Describe how to use this algorithm to find a longest-path in G. Afterwards, count up how many calls to the LP subroutine you made. Is it possible to make do with a single call? Give convincing evidence to support your answer.

Problem 39. A ***circular shift*** of a string $P = \alpha\beta$ is formed by taking a suffix β and concatenating a prefix α. For example, let $P = abcd$. Then the circular shifts of P are: *abcd, bcda, cdab,* and *dabc*.

(a) Given two strings S and T both of length n, write an algorithm to determine whether S is a circular shift of T in time $O(n)$. (*Hint: Use a linear-time exact match algorithm as a subroutine; this problem has a simple and elegant solution.*)

(b) Now given a pattern P of length n and a text T of length m, determine if any circular shift of P occurs in T in $O(n + m)$. *(Hint: Use information the Z-algorithm produces on P and T, and on P and T written backwards.)*