Instructions: The exam has this cover page then four more pages. Please write on the front side of pages only. Make your writing clear and dark—if we can't read it, it's wrong!

A reminder that you may not sit next to any partner or friend (meaning: immediately to the left, right, rear, or diagonal).

Anticipated grading (subject to change): 10 points each for problems 1–9; 40 points for problem 10, based on the number of correct responses in excess of a half.

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1. Let $N = 10291 = 41 \cdot 251$ be the product of two primes. Compute 7^{20000} (mod 10291).

$$\varphi(10291) = 40.250 = 10000, so$$

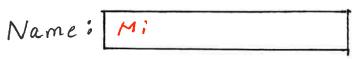
$$\frac{20000}{7} = (7^{10000})^2 = (7^0)^2 = 1 \quad (mod N)$$

2. A trapdoor permutation generator \mathcal{F} is a probabilistic algorithm that, on input of a security parameter k, outputs a pair $(-f, -g) \leftarrow \mathcal{F}(k)$. What's the meaning of those underscores? What's the difference between -f and f?

3. Let N = pq be the product of distinct 200-digit primes, and let $e, d \in \mathbb{Z}_N^*$ be inverses of one another in $\mathbb{Z}_{\phi(N)}^*$. Suppose you **sign directly with RSA**, signing $M \in \mathbb{Z}_N^*$ by $\sigma = M^d \pmod{N}$. Give an adversary $\mathcal{A}^{\mathrm{Sign}_{(N,d)}(\cdot)}(N,e)$ that forges M = 77.

Hint: ask for the signatures of two messages, then output your forgery.

4. The **computational Diffie-Hellman assumption** (CDH) says that doing *what* is hard?

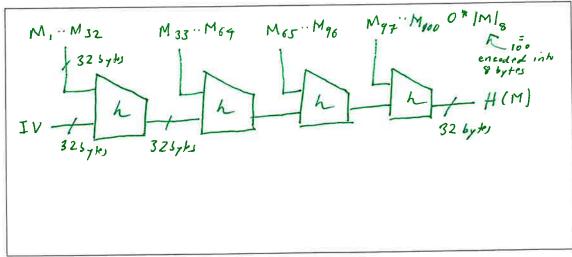


5. Suppose you encrypt with a substitution cipher $\Sigma = (\mathcal{K}, \mathcal{E}, \mathcal{D})$. Key generator \mathcal{K} outputs the description of a random permutation π —Perm(8) specifies a permutation on bytes. The encryption of an n-byte plaintext $M_1 \cdots M_n$ is $\mathcal{E}_{\pi}(M_1 \cdots M_n) = \pi(M_1) \cdots \pi(M_n)$. Now ind-break Σ : Specify an adversary \mathcal{A} whose ind-advantage $\mathbf{Adv}_{\Sigma}^{\mathrm{ind}}(\mathcal{A}) = \Pr[\pi$ —Perm(8) : $\mathcal{A}^{\mathcal{E}_{\pi}(\cdot)} \Rightarrow 1] - \Pr[\pi$ —Perm(8) : $\mathcal{A}^{\mathcal{E}_{\pi}(0|\cdot|)} \Rightarrow 1]$ is large.

Ask oracle for the encryption of AB for any distinct byles A and B (like A = 08, B = 18). Let XY be the response, where |X|=|Y|=8. If X = 1 there return 0.

Simple adversary. Don't ask more than two queries.

6. Recall the Merkle-Damgård construction for making a cryptographic hash function H from a compression function h. Draw a picture that shows what happens when you hash a 100-byte message $M = M_1 M_2 \cdots M_{100}$. Assume that h that maps 64 bytes to 32 bytes. Assume that length annotation (required for Merkle-Damgård) is done by encoding |M| in the last 8 bytes.



7. Define a blockcipher $E: \{0,1\}^{128} \times \{0,1\}^{128} \to \{0,1\}^{128}$ (make sure it *is* a blockcipher) that is **perfectly secure** (prp-advantage of 0) if the adversary asks **one** query, but is **highly insecure** (prp-advantage near 1) if the adversary asks **two** queries.

$$E_K(X) = \mathcal{K} \oplus \mathcal{X}$$

nok: = \mathcal{X} is instruct with one gray;
= \mathcal{K} is not a Slockcipher



8. Let's use Lamport's scheme (lecture 9F) for a one-time, hash-based signature. Assume a hash function H that returns 32 bytes. Suppose the message you will sign is a one byte $M = m_1 m_2 m_3 m_4 m_5 m_6 m_7 m_8$. The public key and secret key will be

$$pk = H(B_1) H(B_2) \cdots H(B_8)$$
 where A_1, \cdots, A_8 , $B_1, \cdots, B_8 \leftarrow \{0_1\}^{8.32}$ $B_1, \cdots, B_8 \leftarrow \{0_1\}^{8.32}$

$$sk = \beta_1, \dots, \beta_8$$

The signature of M = 00001111 will be

9. Cross out and fix (reword) anything that's particularly problematic in the following. Then explain why you made the adjustment you did.

A collision-resistant hash function (also called a collision-

intractable hash function) is a function $H: \{0,1\}^* \to \{0,1\}^n$ with nobody knows

the property that there are no strings M and M' in the domain of H

such that $M \neq M'$ yet H(M) = H(M').

Explanation:

By The PHP, lots of collisions exist — it's just that we don't know any. Us dumb humans, That is.

